Bi-level optimal control method and application

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Introduction

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Optimal control problem

We consider the following Optimal Control Problem in a general Lagrange form:

$$
\text{(OCP)} \qquad \begin{cases} \displaystyle\min_{x,u} \int_{t_0}^{t_f} f^0\big(t, x(t), u(t)\big)\, \mathrm{d}t, \\ \text{s.t. } \dot{x}(t) = f\big(t, x(t), u(t)\big), & t \in [t_0, t_f] \text{ a.e.}, \\ \qquad u(t) \in \mathrm{U}(t), & t \in [t_0, t_f], \\ \qquad c\big(x(t_0), x(t_f)\big) = 0, \end{cases}
$$

where:

- $f^0 \colon \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $f \colon \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ are C^1,
- $t_0 < t_f$ are fixed,
- $\mathrm{U}(t) \subset \mathbb{R}^m$ is a nonempty closed set for every $t \in [t_0, t_f]$, with regularity assumptions,[1]
- $c \colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^p$ is C^1, with $p \leq 2n$ and is a submersion on $c^{-1}(\{0\})$, *i.e.* $c'(a, b)$ is surjective for all pair (a, b) such that $c(a, b) = 0$.

[1](cf. [Cesari, 1983, Chapter 4.2, Remark 5] for more information)

Pontryagin's Maximum Principle

If (x, u) is solution of (OCP), there exists $\lambda \in \mathbb{R}^p$, the costate $p \in \mathrm{AC}([t_0, t_f], \mathbb{R}^n)$ and $p^0 \leq 0$ such that $(p, p^0) \neq 0$, the Hamilton's dynamic is satisfied:

$$\begin{aligned} \dot{x}(t) &= \nabla_p h\big(t, x(t), p(t), u(t)\big) && t \in [t_0, t_f] \text{ a.e.}, \\ \dot{p}(t) &= -\nabla_x h\big(t, x(t), p(t), u(t)\big) && t \in [t_0, t_f] \text{ a.e.}, \end{aligned}$$

the maximisation condition is satisfied:

$$h\left(t, x(t), p(t), u(t)\right) = \max_{w \in \mathrm{U}(t)} h\left(t, x(t), p(t), w\right) \qquad t \in [t_0, t_f] \text{ a.e.},$$

and the transversality condition is satisfied:

$$\begin{pmatrix} -p(t_0) \\ p(t_f) \end{pmatrix} - c'(x(t_0), x(t_f))^\top \lambda = 0.$$

where $h(t, x, p, u) = p^0\, f^0(t, x, u) + \big(p \,\big|\, f(t, x, u)\big)$ is the *pseudo-Hamiltonian*.

Extremals and BC-extremals

Remark

The transversality conditions can be written without λ *by*

$$c^\star\big(z(t_0), z(t_f)\big) := B_c\big(x(t_0), x(t_f)\big)^\top \begin{pmatrix} -p(t_0) \\ p(t_f) \end{pmatrix} = 0,$$

where B_c *is a matrix where each column is a vector of a basis of* $\mathrm{Ker}\ c'(x_0, x_f)$.

An *extremal* is a pair $z = (x, p) \in \mathrm{AC}([t_0, t_f], \mathbb{R}^n) \times \mathrm{AC}([t_0, t_f], \mathbb{R}^n)$ such that it exists $u \in \mathrm{L}^\infty([t_0, t_f], \mathbb{R}^m)$ such that the Hamilton's dynamic and the maximisation condition are satisfied.

A *BC-extremal* is an extremal which satisfy the boundary conditions: the initial and final state constraints given by c, and the initial and final costate constraints, given by $c^\star$.

An extremal is said *normal* if $p^0 < 0$ and *abnormal* if $p^0 = 0$.

Main assumptions

We consider that

- (H1): All the extremals are supposed to be normal and we fix $p^0 = -1$ (by homogeneity of (p^0, p)),
- (H2): The maximized Hamiltonian

$$H(t,z) = \max_{u \in \mathrm{U}(t)} h(t,z,u)$$

is C^1, with $z = (x, p)$ in a neighborhood of a given reference extremal.

Under theses assumptions, the Hamiltonian vector field is defined by

$$\vec{H}(t,z) = \big(\nabla_p H(t,z), -\nabla_x H(t,z)\big),$$

and we get the following proposition

Proposition 1 ([Agrachev and Sachkov, 2004], Proposition 12.1)

$z = (x, p)$ is an extremal of (OCP) if and only if $\dot{z}(t) = \vec{H}(t, z(t))$.

Main idea of the simple shooting method

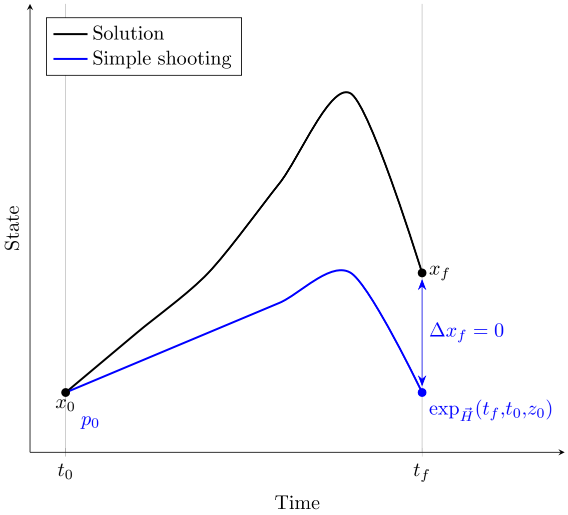

Figure: Illustration of the simple shooting method, where $n = 1$ and a fixed initial and final state (x_0, x_f).

Simple shooting method

The Pontryagin maximum leads to the resolution of the following problem

$$\text{(TPBVP)} \qquad \left\{ \begin{array}{l} z_f = \exp_{\vec{H}}(t_f, t_0, z_0), \\ g(z_0, z_f) = 0, \end{array} \right.$$

where the exponential map $\exp_{\vec{H}}(t_f, t_0, z_0)$ of a field $\vec{H}$ is the solution at time t_f of the Cauchy problem

$$\forall t \in [t_0, t_f],\ \dot{z}(t) = \vec{H}\left(t, z(t)\right),\ z(t_0) = z_0,$$

and g is the state and costate initial and final constraints, defined by

$$g(z_0, z_f) = \begin{pmatrix} c(x_0, x_f) \\ c^\star(z_0, z_f) \end{pmatrix}, \quad \text{where} \quad \begin{array}{l} z_0 = (x_0, p_0) \\ z_f = (x_f, p_f) \end{array}.$$

The simple shooting methods aim to find a zero of the following shooting function

$$\begin{array}{rcccc} S_s & : & \mathbb{R}^{2n} & \longrightarrow & \mathbb{R}^{2n} \\ & & z_0 & \longmapsto & g\left(z_0, \exp_{\vec{H}}(t_f, t_0, z_0)\right). \end{array}$$

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Goals

The application is an industrial problem and the method need to be:

- fast,
- robust,
- computationally efficient.

Figure: Master controller.

Main idea of the multiple shooting method

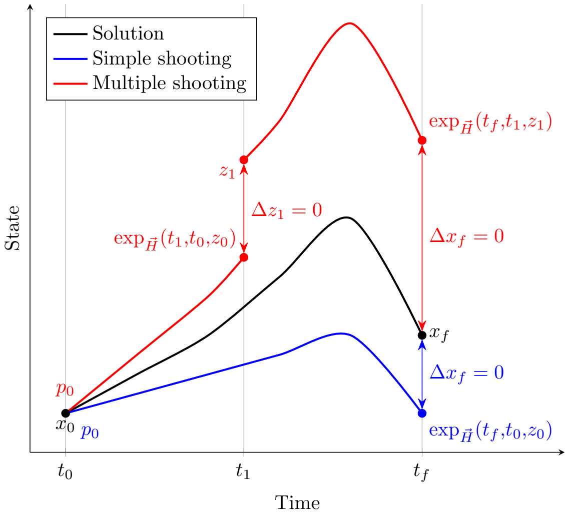

Figure: Illustration of the simple and the multiple shooting method, where $n = 1$ and a fixed initial and final state (x_0, x_f).

Multiple shooting method

The time interval $[t_0, t_f]$ is decomposed into $\Delta_i = [t_i, t_{i+1}], i \in \mathbb{N}_N$, where $t_0 < t_1 < \cdots < t_N < t_{N+1} = t_f$ and $\mathbb{N}_N = \{0, \ldots, N\}$.

(TPBVP) is transformed to

(MPBVP) $$\begin{cases} \forall i \in \mathbb{N}_{N-1}, \quad z_{i+1} = \exp_{\vec{H}}(t_i, t_{i+1}, z_i), \\ g\big(z_0, \exp_{\vec{H}}(t_{N+1}, t_N, z_N)\big) = 0. \end{cases}$$

The corresponding shooting function $S_m \colon \mathbb{R}^{2n(N+1)} \to \mathbb{R}^{2n(N+1)}$ is defined by

$$S_m(z_0, \ldots, z_N) = \begin{pmatrix} \exp_{\vec{H}}(t_1, t_0, z_0) - z_1 \\ \vdots \\ \exp_{\vec{H}}(t_N, t_{N-1}, z_{N-1}) - z_N \\ g\big(z_0, \exp_{\vec{H}}(t_{N+1}, t_N, z_N)\big) \end{pmatrix}.$$

S_m is known to be less sensitive to the initial guess than S_s [Bock and Plitt, 1984].

Goals

Comparated to the simple shooting method, the multiple shooting one is

- ✓ faster,
- ✓ more robust,
- × computationally equivalent.

Goal 1: See the multiple shooting method with a different point of view.

Goal 2: Propose a new optimal control method.

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Main idea of the bilevel formulation

Example: Cycling race.

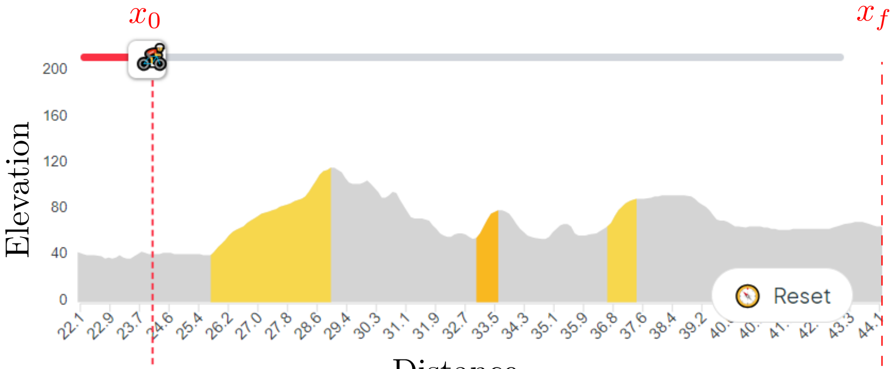

Figure: Part of the Paris-Roubaix race [2]

[2]https://bikespot.fr/en/routes/1-paris-roubaix#readElevation

Example: Cycling race.

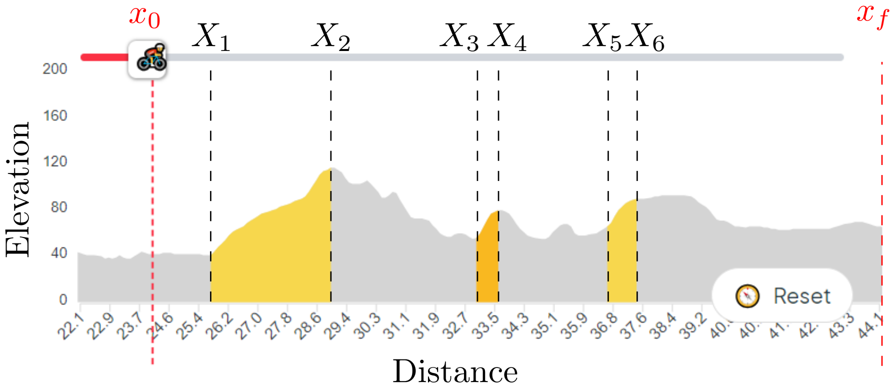

Figure: Part of the Paris-Roubaix race [2]

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Intermediate optimal control problems

For all $i \in \mathbb{N}_N$, the intermediate optimal control problems are defined by

$$(\mathrm{OCP}_{i,a,b}) \qquad \begin{cases} V_i(a,b) := \displaystyle\min_{x,u} J_i(x,u) \\ \text{s.t. } \dot{x}(t) = f\big(t, x(t), u(t)\big), & t \in \Delta_i \text{ a.e.}, \\ \qquad u(t) \in \mathrm{U}(t), & t \in \Delta_i, \\ \qquad x(t_i) = a, \quad x(t_{i+1}) = b, \end{cases}$$

where V_i corresponds to the *value function*. The cost J_i is defined by

$$J_i(x,u) = \int_{t_i}^{t_{i+1}} f^0(t, x(t), u(t))\,\mathrm{d}t,$$

and let $\mathcal{S}_i(a,b)$ the set of solutions of $(\mathrm{OCP}_{i,a,b})$.

Bilevel formulation of (OCP)

(OCP) can be formulate into the equivalent form

$$
\text{(BOCP)} \qquad \left\{ \begin{array}{l} \min\limits_{X} V(X) := \sum\limits_{i=0}^{N} V_i(X_i, X_{i+1}) \\ \text{s.t.} \quad X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \end{array} \right.
$$

where $\mathcal{X}$ is the set of admissible intermediate states $X = \big(X_0, \ldots, X_{N+1}\big)$.

Bilevel formulation of (OCP)

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$$\begin{cases} \min_X V(X) := \sum_{i=0}^{N} V_i(X_i, X_{i+1}) \\ \text{s.t.} \quad X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \end{cases}$$

where $\mathcal{X}$ is the set of admissible intermediate states $X = \big(X_0, \dots, X_{N+1}\big)$.

Let's remark that for all $i \in \mathbb{N}_N$, for all (a, b) admissible, and for all $(x_i, u_i) \in \mathcal{S}_i(a, b)$, we have

$$J_i(x_i, u_i) = V_i(a, b),$$

and so, (BOCP) can be seen as bilevel optimal control problem [Aussel and Svensson, 2020]

$$\begin{cases} \min_X \min_{x,u} \sum_{i=0}^{N} J_i(x_i, u_i) \\ \text{s.t.} \quad X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \\ \qquad \forall i \in \mathbb{N}_N, \quad (x_i, u_i) \in \mathcal{S}_i(X_i, X_{i+1}). \end{cases}$$

Necessary conditions

Assumption

The function V is differentiable at the solution of (BOCP)

Thanks to KKT conditions, if X is solution of (BOCP), there exists $\lambda \in \mathbb{R}^p$ such that

$$\text{(NCBOCP)} \qquad \begin{cases} \nabla_X L(X, \lambda) = 0, \\ X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \end{cases}$$

where $L\colon (\mathbb{R}^n)^{N+1} \times \mathbb{R}^p \to \mathbb{R}$ is the Lagrangian associated to (BOCP), defined by

$$L(X, \lambda) = V(X) - \left(\lambda \,\middle|\, c(X_0, X_{N+1})\right).$$

Using the expressions of L and V, we have

$$\text{(NCBOCP)} \Leftrightarrow \begin{cases} \begin{pmatrix} \nabla_a V_0(X_0, X_1) \\ \nabla_b V_N(X_N, X_{N+1}) \end{pmatrix} - c'(X_0, X_{N+1})^\top \lambda = 0, \\ \nabla_b V_{i-1}(X_{i-1}, X_i) + \nabla_a V_i(X_i, X_{i+1}) = 0, \quad \forall i \in \{1, \dots, N\}, \\ c(X_0, X_{N+1}) = 0, \quad X \in \mathcal{X}, \end{cases}$$

Main theorem

Theorem 1

Given (a, b) admissible, we consider a particular case (OCP_*) of (OCP) in which $c(x(t_0), x(t_f)) = (x(t_0) - a, x(t_f) - b)$ (initial and final conditions imposed):

$$
(\mathrm{OCP}_*) \qquad \begin{cases} V_*(a,b) := \displaystyle\min_{x,u} \int_{t_0}^{t_f} f^0\left(t, x(t), u(t)\right)\, \mathrm{d}t, & \\ \text{s.t. } \dot{x}(t) = f\left(t, x(t), u(t)\right) & t \in [t_0, t_f] \text{ a.e.}, \\ \quad u(t) \in \mathrm{U}(t), & t \in [t_0, t_f], \\ \quad x(t_0) = a, \quad x(t_f) = b. & \end{cases}
$$

The value function $V_*(a, b)$ of (OCP_*) is assumed to be differentiable at (a, b). Then, if (x, u) is a solution of (OCP_*) with (x, p) the associated normal BC-extremal, we have:

$$
\nabla V_*\left(x(t_0), x(t_f)\right) = \left(-p(t_0), p(t_f)\right).
$$

Idea of proof

We want to prove that

$$\nabla_a V_*\big(x(t_0), x(t_f)\big) = -p(t_0), \tag{1}$$
$$\nabla_b V_*\big(x(t_0), x(t_f)\big) = p(t_f). \tag{2}$$

(1) is a classical result [Bokanowski et al., 2021]. To prove (2), we transform (OCP_*) into

$$(\mathrm{ROCP}_*) \quad \begin{cases} V_R(b, a) := \min_{\hat{x}, \hat{u}} \int_{t_0}^{t_f} f^0(\phi(t), \hat{x}(t), \hat{u}(t))\, \mathrm{d}t, \\ \text{s.t. } \dot{\hat{x}}(t) = -f(\phi(t), \hat{x}(t), \hat{u}(t)), & t \in [t_0, t_f] \text{ a.e.}, \\ \quad \hat{u}(t) \in \mathrm{U}(\phi(t)), & t \in [t_0, t_f], \\ \quad \hat{x}(t_0) = b, \quad \hat{x}(t_f) = a, \end{cases}$$

where the reverse time transformation $\phi \colon [t_0, t_f] \to [t_0, t_f]$ is defined by

$$\phi(t) = t_f + t_0 - t.$$

Idea of proof

Using the classical transformation

$$\theta_R(x, p, u) = (x \circ \phi, -p \circ \phi, u \circ \phi)$$

and denoting $(\hat{x}, \hat{p}, \hat{u}) = \theta_R(x, p, u)$, (OCP_*) is equivalent to (ROCP_*):

$$\begin{aligned} & (x, p) \text{ is a BC-extremal associated to } (x, u) \text{ solution of } (\text{OCP}_*) \\ \Longleftrightarrow\ & (\hat{x}, \hat{p}) \text{ is a BC-extremal associated to } (\hat{x}, \hat{u}) \text{ solution of } (\text{ROCP}_*). \end{aligned}$$

Since (ROCP_*) has the same form as (OCP_*) and the value function V_R is differentiable at (b, a), we can apply (1) to (ROCP_*):

$$\begin{aligned} \nabla_b V_*(x(t_0), x(t_f)) &= \nabla_b V_R(\hat{x}(t_0), \hat{x}(t_f)) \\ &= -\hat{p}(t_0) \\ &= -(-p \circ \phi)(t_0) \\ &= p(t_f). \end{aligned}$$

□

Necessary conditions

The necessary optimality conditions of (BOCP)

$$
\text{(NCBOCP)} \begin{cases} \begin{pmatrix} \nabla_a V_0(X_0, X_1) \\ \nabla_b V_N(X_N, X_{N+1}) \end{pmatrix} - c'(X_0, X_{N+1})^\top \lambda = 0, \\ \nabla_b V_{i-1}(X_{i-1}, X_i) + \nabla_a V_i(X_i, X_{i+1}) = 0, \quad \forall i \in \{1, \ldots, N\}, \\ c(X_0, X_{N+1}) = 0, \quad X \in \mathcal{X}, \end{cases}
$$

become, by using the Theorem 1

$$
\text{(NCBOCP)} \Leftrightarrow \begin{cases} \forall i \in \mathbb{N}_N,\ \exists\, z_i = (x_i, p_i) \text{ a BC-extremal associated} \\ \text{to a solution } (x_i, u_i) \text{ of } (\mathrm{OCP}_{i, X_i, X_{i+1}}), \\ \begin{pmatrix} -p_0(t_0) \\ p_N(t_{N+1}) \end{pmatrix} - c'(X_0, X_{N+1})^\top \lambda = 0, \\ \forall i \in \{1, ..., N\},\ p_{i-1}(t_i) - p_i(t_i) = 0, \\ c(X_0, X_{N+1}) = 0. \end{cases}
$$

Necessary conditions

Replacing (x_i, u_i) solution to $(\mathrm{OCP}_{i,X_i,X_{i+1}})$ by the associated necessary conditions of optimality, we get

$$
(\mathrm{NCBOCP}) \Rightarrow \begin{cases}
\forall i \in \mathbb{N}_N,\ \exp_{\vec{H}}\left(t_{i+1}, t_i, z_i(t_i)\right) = z_i(t_{i+1}), \\
\forall i \in \mathbb{N}_N,\ x_i(t_i) - X_i = 0, \\
\forall i \in \mathbb{N}_N,\ x_i(t_{i+1}) - X_{i+1} = 0, \\
\begin{pmatrix} -p_0(t_0) \\ p_N(t_{N+1}) \end{pmatrix} - c'(X_0, X_{N+1})^\top \lambda = 0, \\
\forall i \in \{1, \dots, N\},\ p_{i-1}(t_i) - p_i(t_i) = 0, \\
c(X_0, X_{N+1}) = 0,
\end{cases}
$$

$$
\Leftrightarrow (\mathrm{MPBVP}).
$$

Commutative diagram

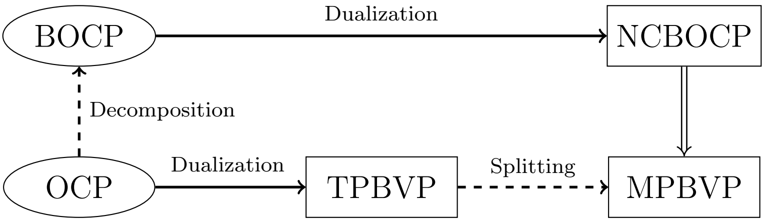

Figure: Commutative diagram from (OCP) to (MPBVP).

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Main idea

Let's assume that the value functions V_i are known a priori.
We have to solve an optimization problem

$$\begin{cases} \min_X V(X) := \sum_{i=0}^{N} V_i\left(X_i, X_{i+1}\right) \\ \text{s.t. } X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \end{cases}$$

to get the optimal intermediate states $X^* = \left(X_0^*, \dots, X_{N+1}^*\right)$,

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to get the optimal intermediate states $X^* = \left(X_0^*, \dots, X_{N+1}^*\right)$, and $N+1$ independent optimal control problems

$$\begin{cases} \min\limits_{x,u} \int_{t_i}^{t_{i+1}} f^0\left(t, x(t), u(t)\right) \mathrm{d}t \\ \text{s.t. } \dot{x}(t) = f\left(t, x(t), u(t)\right), & t \in \Delta_i \text{ a.e.}, \\ \qquad u(t) \in \mathrm{U}(t), & t \in \Delta_i, \\ \qquad x(t_i) = X_i^*, \quad x(t_{i+1}) = X_{i+1}^*, \end{cases}$$

where $\left(X_i^*, -\nabla_a V_i(X_i^*, X_{i+1}^*)\right)$ is a zero of the associated simple shooting function.

Proposed approach

The proposed approach is based on an approximation C_i of the value function V_i.
We propose to solve an optimization problem

$$(\text{Macro}) \qquad \begin{cases} \min_X C(X) := \sum_{i=0}^{N} C_i\left(X_i, X_{i+1}\right) \\ \text{s.t. } X \in \mathcal{X}, \quad c(X_0, X_{N+1}) = 0, \end{cases}$$

to get the "optimal" intermediate states $\hat{X} = \left(\hat{X}_0, \ldots, \hat{X}_{N+1}\right)$,

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(Macro)
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to get the "optimal" intermediate states $\hat{X} = \left(\hat{X}_0, \dots, \hat{X}_{N+1}\right)$, and $N+1$ independent optimal control problems

(Micro)
$$\begin{cases} \min_{x,u} \int_{t_i}^{t_{i+1}} f^0\left(t, x(t), u(t)\right) \mathrm{d}t \\ \text{s.t. } \dot{x}(t) = f\left(t, x(t), u(t)\right), & t \in \Delta_i \text{ a.e.}, \\ \quad u(t) \in U(t), & t \in \Delta_i, \\ \quad x(t_i) = \hat{X}_i, \quad x(t_{i+1}) = \hat{X}_{i+1}, \end{cases}$$

where $\left(\hat{X}_i, -\nabla_a C_i(\hat{X}_i, \hat{X}_{i+1})\right)$ is not necessary a zero of the associated simple shooting function.

Error analysis

The optimal cost is $V(X^*)$, and the (Macro)-(Micro) cost is $V(\hat{X})$. How to control the error of the proposed method $|V(X^*) - V(\hat{X})|$?

Proposition 2

If there exists $\varepsilon \geq 0$ such that for all $i \in N_N$ and for all (a, b) admissible

$$|V_i(a, b) - C_i(a, b)| \leq \frac{\varepsilon}{2(N+1)},$$

then we have

$$|V(X^*) - V(\hat{X})| \leq \varepsilon.$$

How to create C_i, the approximation of V_i ?

Database of optimal values: method 1

To approximate V_i, we need to create a database $\mathbb{D}_i$ of optimal transition values

$$\mathbb{D}_i = \Big\{ (a, b, c) \;\Big|\; (a, b) \text{ admissible and } V_i(a, b) = c \Big\}.$$

<u>Method 1:</u> for a given set of initial and final admissible state (a, b), compute $V_i(a, b)$, this means solving the optimal control $(\mathrm{OCP}_{i,a,b})$.

With the simple shooting method, it leads to the resolution of $S_{i,a,b}(z_0) = 0$, where $S_{i,a,b} \colon \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is the associated simple shooting function defined by

$$S_{i,a,b}(z_0) = \begin{pmatrix} \pi_x(z_0) - a \\ \pi_x\big(\exp_{\vec{H}}(t_{i+1}, t_i, z_0)\big) - b \end{pmatrix},$$

where π_x is the classical state projection $\pi_x(x, p) = x$.

Database of optimal values: method 2

Method 2: For a discretization of initial state and costate $z_0 = (x_0, p_0)$, calculate $z_f = (x_f, p_f) = \exp_{\vec{H}}(t_i, t_{i+1}, z_0)$ and the associated cost c. We have [3]

$$V_i(x_0, x_f) = c.$$

Advantages: No need to solve $S_{i,a,b}(z_0) = 0$: less computation and faster method.
Disadvantages: No control on the database repartition.

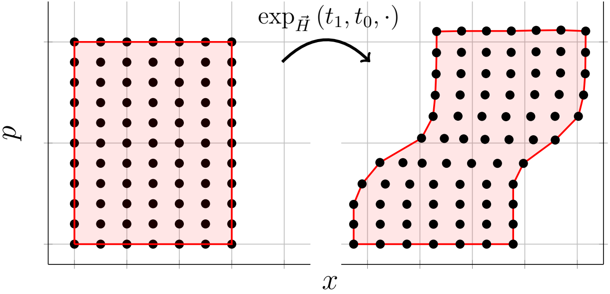

Figure: Hamiltonian bend for the first time interval for the studied application.

[3] under some assumption on $p_0 \mapsto \pi_x(\exp_{\vec{H}}(t_{i+1}, t_i, (x_0, p_0)))$

Cost transition functions C_i

If the database $\mathbb{D}_i$ is made by method 2, the transition cost C_i can be modeled by a simple smooth neural network.

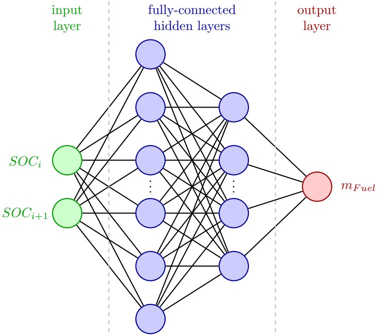

Figure: Schema of the network.

Architecture: 2 hidden layers (16/8 neurons), tanh and sigmoïd activations.

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Cycle

We consider an Hybrid Electric Vehicle (HEV) on a predefined cycle, i.e. speed and slope trajectories are prescribed.

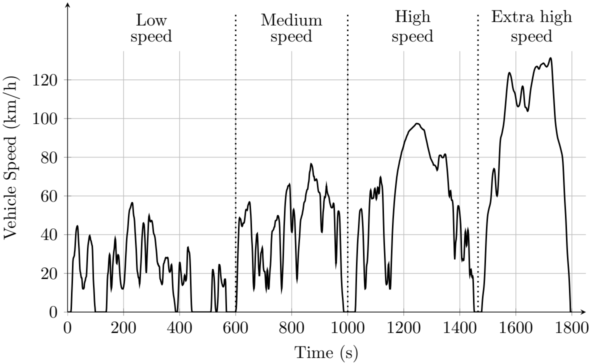


Figure: Worldwide harmonized Light vehicles Test Cycle (WLTC).

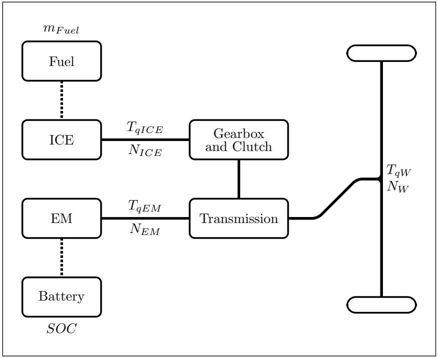
Requested wheels torque $T_{qW}(t)$ and rotation speed $N_W(t)$ are obtained with the information of our vehicle (mass, wheel diameter, aerodynamic coefficient. . .).

Static model of HEV

Inputs of our static model:

Name	Description	Unit
Cost		
m_{Fuel}	Fuel consumption	g
State		
SOC	Battery state of charge	
Commands		
$Gear$	Gearbox selector	
T_{qICE}	ICE torque	N.m
External inputs		
T_{qW}	Wheels torque	N.m
N_W	Wheels rotation speed	RPM

Figure: Schema of the selected HEV.



Outputs: $\dot{m}_{Fuel}$ and $\dot{SOC}$, where $\dot{}$ stands for $\dfrac{\mathrm{d}}{\mathrm{d}t}$.

HEV torque split and gear shift problem

The Hybrid Electric Vehicle torque split and gear shift problem can be formulated as the same form as (OCP):

$$
(\mathrm{OCP}):\begin{cases}
\displaystyle\min_{x,u}\int_{t_0}^{t_f} f^0\big(t,x(t),u(t)\big)\mathrm{d}t & \\
\text{s.t. } \dot{x}(t)=f\big(t,x(t),u(t)\big) & t\in[t_0,t_f] \text{ a.e.},\\
\quad u(t)\in \mathrm{U}(t) & t\in[t_0,t_f],\\
\quad c\big(x(t_0),x(t_f)\big)=\begin{pmatrix} x(t_0)-x_0\\ x(t_f)-x_f\end{pmatrix}=0, &
\end{cases}
$$

where:

- $x = SOC$ (State Of Charge),
- $u = \big(T_{qICE}, Gear\big)$,
- f^0 is the instantaneous fuel consumption function,
- f describes the instantaneous evolution of the state of charge.

The assumptions of (OCP) are satisfied.

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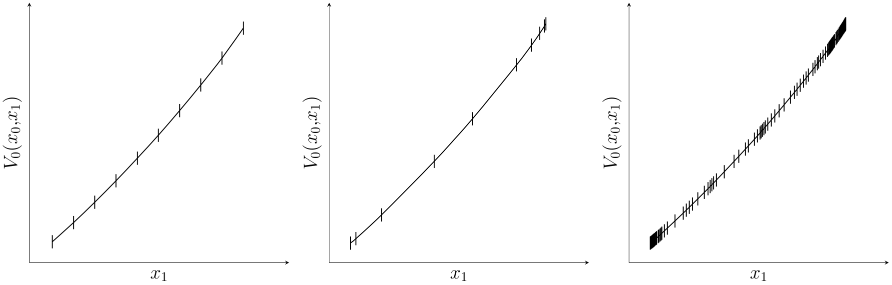
Database of optimal values: Comparison

Mean number of iterations to solve the shooting problem[4]: 11.2.



(a) Method 1, 10 points. (b) Method 2, 10 points. (c) Method 2, 112 points.

Figure: Value function data, created with the method 1 with 10 points (a), and with the method 2 with 10 points (b) and 112 points (c), on the first time interval ($i = 0$), and with a fixed initial state.

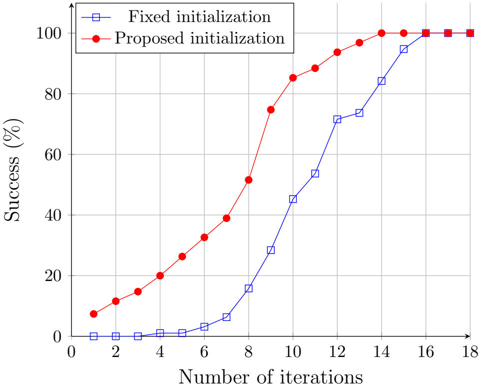
[4] with fixed initialization, on 475 experiments on the first time interval with different initial and final state

(Micro) problems resolution

Thanks to Theorem 1, the couple

$$\left(\hat{X}_i, -\nabla_a C_i(\hat{X}_i, \hat{X}_{i+1})\right)$$

is a natural initial guess to find a zero of the associated shooting function.



Results

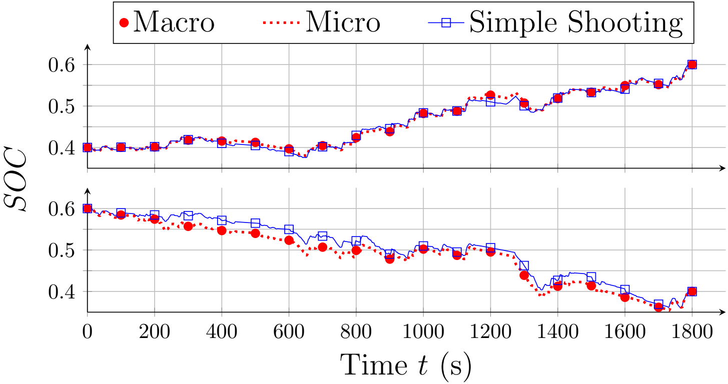


Figure: State trajectories of the simple shooting and the bi-level methods.

Associated cost error: 0.34g (0.039%) and 1.71g (0.244%).

Conclusion

We proposed a new method (Macro)-(Micro):

- based on a bilevel decomposition of (OCP),
- strongly linked to the Multiple shooting method ,
- faster than the simple shooting method, due to the parallel computing,
- that need less computation for embedded solution than multiple shooting method,
- with a good initialization of the shooting function,
- with small cost difference.

Perspectives:

- Generalization on multiple cycle: Convolutional Neural Network,
- Generalization of the method with weaker knowledge assumptions.

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