A geometric preconditioner for indirect optimal control method and application to hybrid electric vehicle.

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Introduction

In collaboration with:

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Cycle

We consider a Hybrid Electric Vehicle (HEV) on a predefined cycle, i.e. speed and slope trajectories are prescribed.

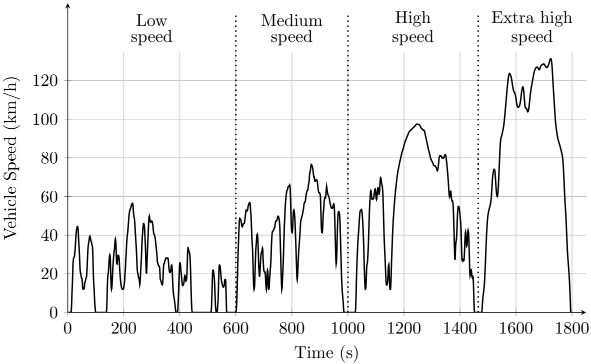


Figure: Worldwide harmonized Light vehicles Test Cycle (WLTC).

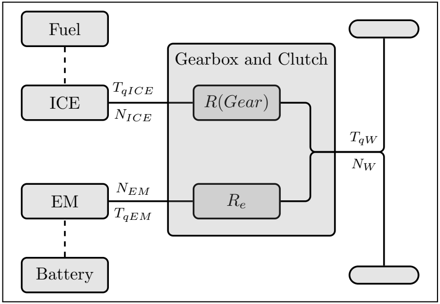
Requested wheels torque $T_{qW}(t)$ and rotation speed $N_W(t)$ are obtained with the information of our vehicle (mass, wheel diameter, aerodynamic coefficient. . .).

Static model of HEV

Inputs of our static model:

Name	Description	Unit
Cost		
m_{Fuel}	Fuel consumption	g
State		
SOC	Battery state of charge	
Commands		
$Gear$	Gearbox selector	
T_{qICE}	ICE torque	N.m
External inputs		
T_{qW}	Wheels torque	N.m
N_W	Wheels rotation speed	RPM

Figure: Schema of the HEV.



Outputs: $\dot{m}_{Fuel}$ and $\dot{SOC}$, where $\dot{}$ stands for $\dfrac{\mathrm{d}}{\mathrm{d}t}$.

HEV torque split and gear shift problem

The HEV torque split and gear shift problem can be formulated as a classical Lagrange optimal control problem

$$
\text{(OCP)} \quad \begin{cases} V(x_0, x_T) = \displaystyle\min_{x,u} \int_{t_0}^{t_f} f^0\big(t, x(t), u(t)\big)\, \mathrm{d}t, & \\ \text{s.t. } \dot{x}(t) = f\big(t, x(t), u(t)\big), & t \in [t_0, t_f] \text{ a.e.}, \\ \qquad u(t) \in \mathrm{U}(t), & t \in [t_0, t_f], \\ \qquad x(t_0) = x_0, \quad x(t_f) = x_T, & \end{cases}
$$

where:

- $x \in \mathrm{AC}([t_0, t_f], \mathbb{R})$ corresponds to the *SOC*,
- $u \in \mathrm{L}^\infty([t_0, t_f], \mathbb{R}^2)$ corresponds to the pair $\big(T_{qICE}, \mathit{Gear}\big)$,
- functions f^0 and f are $\mathcal{C}^1$ w.r.t. x and u,
- $\mathrm{U}(t) \subset \mathbb{R}^2$ is a nonempty closed set for every $t \in [t_0, t_f]$, with regularity assumptions.[1]

[1](cf. [Cesari, 1983, Chapter 4.2, Remark 5] for more information)

Assumptions from previous work

Motivated by [Cots et al., 2023a],

- we only consider the first 100s of the cycle ($[t_0, t_f] = [0, 100]$),
- we have constructed a database $\mathbb{D}$ of the value function V evaluations by an efficient method[2],
- we have trained a neural network C on $\mathbb{D}$ to approximate V.

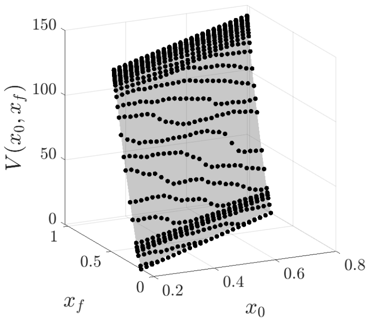


Figure: The points correspond to $\mathbb{D}$ and the surface to the trained neural network C.

[2] cf. [Cots et al., 2023b] for more information

Augmented system

We propose to consider the augmented formulation of (OCP)

$$
\text{(AOCP)} \qquad \begin{cases} \min\limits_{\hat{x},u} x^0(t_f) \\ \text{s.t. } \dot{\hat{x}}(t) = \hat{f}\big(t, \hat{x}(t), u(t)\big) & t \in [t_0, t_f] \text{ a.e.}, \\ \qquad u(t) \in \mathrm{U}(t) & t \in [t_0, t_f], \\ \qquad \hat{x}(t_0) = \hat{x}_0, \quad x(t_f) = x_f, \end{cases}
$$

where $\hat{f} \colon \mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is the augmented system

$$
\hat{f}(t, \hat{x}, u) = \big(f^0(t, x, u), f(t, x, u)\big)
$$

and where $\hat{x} = (x^0, x)$ corresponds to the cost-state pair, with $\hat{x}_0 = (0, x_0)$.

Pontryagin's Maximum Principle

If $(\hat{x}, u)$ is solution of (AOCP), there exists a non trivial augmented costate $\hat{p} = (p^0, p) \in \mathrm{AC}([t_0, t_f], \mathbb{R}^2) \neq 0$ with $p^0 \leq 0$ such that the Hamilton's dynamic is satisfied for almost every $t \in [t_0, t_f]$

$$\begin{cases} \dot{\hat{x}}(t) = \quad \nabla_{\hat{p}} h\big(t, \hat{x}(t), \hat{p}(t), u(t)\big), \\ \dot{\hat{p}}(t) = -\nabla_{\hat{x}} h\big(t, \hat{x}(t), \hat{p}(t), u(t)\big), \end{cases} \tag{1}$$

as well as the maximization condition for almost every $t \in [t_0, t_f]$

$$h\big(t, \hat{x}(t), \hat{p}(t), u(t)\big) = \max_{w \in \mathrm{U}(t)} h\big(t, \hat{x}(t), \hat{p}(t), w\big), \tag{2}$$

where $h(t, \hat{x}, \hat{p}, u) = \big(\hat{p} \,\big|\, \hat{f}(t, \hat{x}, u)\big)$ is the pseudo-Hamiltonian of the augmented system.

Notations

For the following presentation, we denote

$$\hat{x} = (x^0, x) \quad \text{and} \quad \hat{p} = (p^0, p).$$

Moreover, we denote

$$\hat{z} = (\hat{x}, \hat{p}) \quad \text{and} \quad z = (x, p).$$

These notations can be used for absolutely continuous functions or for vectors.

Remark

Since $\hat{f}$ does not depend on the cost x^0, we obtain $\dot{p}^0(\cdot) = 0$ and thus $p^0(\cdot)$ is constant.

Definitions - Extremals

An extremal is a couple $(\hat{z}, u) \in \mathrm{AC}([t_0, t_f], \mathbb{R}^4) \times \mathrm{L}^\infty([t_0, t_f], \mathbb{R}^2)$ which satisfies the Hamilton's dynamic (1) and the maximization condition (2).

A BC-extremal is an extremal which satisfies the boundary conditions given by $\hat{x}(t_0) = \hat{x}_0$ and $x(t_f) = x_T$.

An extremal is said normal$^-$ if $p^0 < 0$, normal$^+$ if $p^0 > 0$ and abnormal if $p^0 = 0$.

Framework

Let us denote $\exp_{\vec{h}}(\hat{z}_0)$ a solution at time t_f of

$$\begin{cases} \dot{\hat{z}}(t) = \vec{h}\left(t, \hat{z}(t), u(t)\right), & t \in [t_0, t_f] \text{ a.e.} \\ h\left(t, \hat{z}(t), u(t)\right) = \max_{w \in \mathrm{U}(t)} h\left(t, \hat{z}(t), w\right), & t \in [t_0, t_f] \text{ a.e.} \\ \hat{z}(t_0) = \hat{z}_0, \end{cases}$$

where $\vec{h}$ is the pseudo-Hamiltonian vector field of the augmented system, defined by

$$\vec{h}(t, \hat{x}, \hat{p}, u) = \left(\nabla_{\hat{x}} h(t, \hat{x}, \hat{p}, u), -\nabla_{\hat{p}} h(t, \hat{x}, \hat{p}, u)\right).$$

Hypothesis 1

The possibly multivalued function $\exp_{\vec{h}}(\hat{x}_0, \hat{p}_0)$ is an application, defined for all $\hat{x}_0 \in \mathbb{R}^2$ and for all non trivial $\hat{p}_0 \in \mathbb{R}^2$.

Simple shooting method

Under the previous hypothesis, the maximum principle leads to the resolution of

$$\text{(TPBVP)} \qquad \begin{cases} \pi_x\big(\exp_{\vec{h}}(\hat{z}_0)\big) = x_T, \\ \pi_{\hat{x}}(\hat{z}_0) = \hat{x}_0, \quad \pi_{p^0}(\hat{z}_0) \leq 0, \end{cases}$$

where $\pi_x(\cdot)$ is the classical x-space projection.

The simple shooting method aims to find a non-trivial zero $\hat{p}_0 = (p^0, p_0)$ of the shooting function

$$\begin{array}{rcccc} S & : & \mathbb{R}^- \times \mathbb{R} & \longrightarrow & \mathbb{R} \\ & & \hat{p}_0 & \longmapsto & \pi_x\big(\exp_{\vec{h}}(\hat{x}_0, \hat{p}_0)\big) - x_T \end{array}$$

Normalization of the shooting function

Let us remark that if $\hat{p}_0 \neq 0$ satisfies $S(\hat{p}_0) = 0$ then for all $k > 0$, $S(k\hat{p}_0) = 0$ (due to homogeneity of BC-extremals on $\hat{p}$).

We propose two normalizations of the shooting function S.

- Method 1: if we assume that the extremals associated to a solution are normal$^-$ ($p^0 < 0$), then we can fix $p^0 = -1$ and consider $S_1 \colon \mathbb{R} \to \mathbb{R}$ defined by

$$S_1(p_0) = S(-1, p_0),$$

- Method 2: without the above assumption, we can fix $\|\hat{p}_0\|_2 = 1$ and consider $S_2 \colon [-1, 1] \to \mathbb{R}$ defined by

$$S_2(p_0) = S\big(\eta(p_0), p_0\big), \quad \text{where} \quad \eta(p_0) = -\sqrt{1 - p_0^2}.$$

A solution of the shooting method is found by a Newton-like solver.

Thanks to [Cots et al., 2023a], a natural initial guess for S_1 is given by

$$p_* = -\nabla_{x_0} C(x_0, x_f).$$

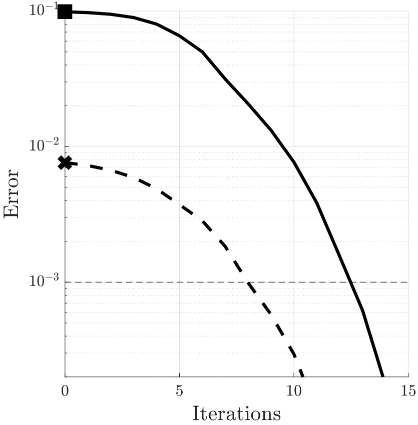


Figure: Evolution of the error $|S_1(\cdot) - x_f|$ w.r.t the number of iterations (with 100 different initial and final states).
——: fixed initialization $p = 500$ (■) - - -: natural initialization p_* (✖)
---: industrial tolerance 10^{-3}

Goal

<u>**Goal**</u>: Reducing the number of iterations of the solver

<u>**Main idea**</u>[3]: Preconditioning method of the shooting function based on

- a geometric interpretation of the costate
- and the Mathieu transformation.

[3] cf. [Cots et al., 2024] for more information

Geometric interpretation of the costate

The proof of the maximum principle is constructive.

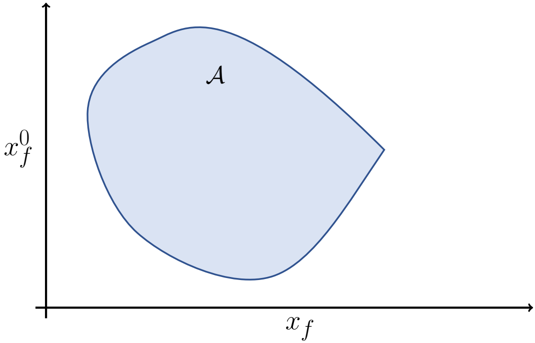


Figure: Illustration of the accessible augmented state set $\mathcal{A}$, which is the set of reachable augmented states $\hat{x}_f = (x_f^0, x_f)$ at t_f from $\hat{x}_0$ at t_0.

Geometric interpretation of the costate

The proof of the maximum principle is constructive. The final augmented costate $\hat{p}_f = \left(p^0, p(t_f)\right)$ is taken in the polar of the proper convex Boltyanskii cone $\mathcal{K}^\circ$.

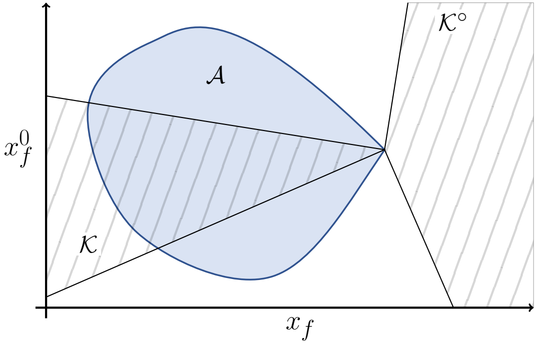


Figure: Illustration of the Botlyanskii cone $\mathcal{K}$ and its polar $\mathcal{K}^\circ$ at an augmented final state $\hat{x}_f \in \partial\mathcal{A}$.

Geometric interpretation of the costate

The proof of the maximum principle is constructive. The final augmented costate $\hat{p}_f = \left(p^0, p(t_f)\right)$ is taken in the polar of the proper convex Boltyanskii cone $\mathcal{K}^\circ$.

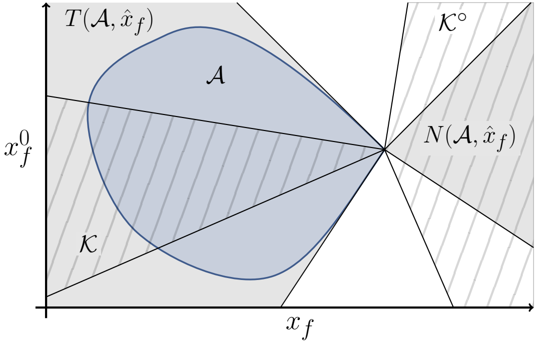
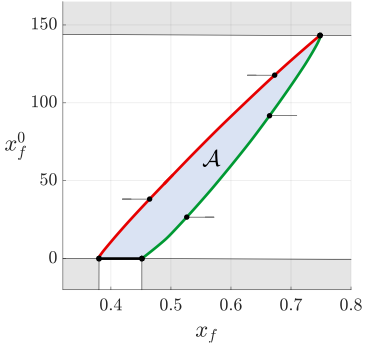
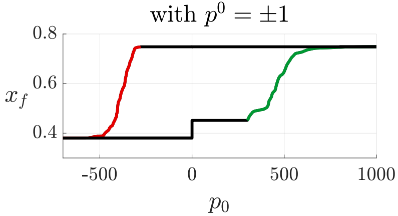


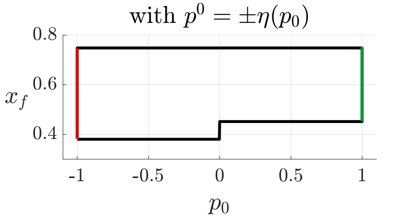
Figure: Illustration of the link between $\mathcal{K}^\circ$ and the normal cone $N\left(\mathcal{A}, \hat{x}_f\right)$ of the set $\mathcal{A}$ at the point $\hat{x}_f$.

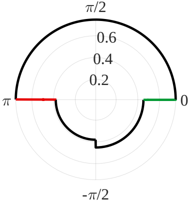
If $\mathcal{A}$ is closed and convex, we can take $\hat{p}(t_f) \in N\left(\mathcal{A}, \hat{x}_f\right)$.

Accessible augmented set and shooting functions

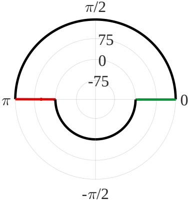








State x_f



Cost x_f^0

Figure: With $x_0 = 0.5$.

Mathieu transformation

A diffeomorphism $\phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ on the augmented state is lifted into a diffeomorphism $\Phi\colon \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2 \times \mathbb{R}^2$ on the augmented state-costate that preserves the Hamiltonian dynamics

$$\Phi(\hat{x}, \hat{p}) = \left(\phi(\hat{x}), J_\phi(\hat{x})^{-\top} \hat{p}\right),$$

which is called Mathieu transformation.

This diffeomorphism transforms $\hat{z} = (\hat{x}, \hat{p})$ into $\hat{w} = (\hat{y}, \hat{q})$:

$$\hat{z} = \begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix} \underset{\Phi^{-1}}{\overset{\Phi}{\rightleftarrows}} \begin{pmatrix} \hat{y} \\ \hat{q} \end{pmatrix} = \hat{w}.$$

Moreover, we denote $\hat{y} = (y^0, y)$ and $\hat{q} = (q^0, q)$.

Construction of the transformation

Main idea: fitting an ellipse on $\partial \mathcal{A}$ and creating the linear diffeomorphism $\phi(\hat{x}) = A\hat{x} + b$ that transforms this ellipse into the unit circle.

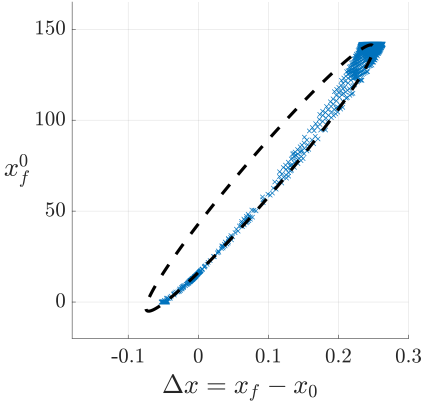


Figure: Original coordinates

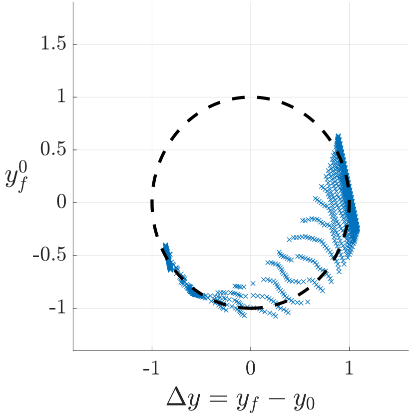


Figure: New coordinates

Construction of the transformation

Main idea: fitting an ellipse on $\partial\mathcal{A}$ and creating the linear diffeomorphism $\phi(\hat{x}) = A\hat{x} + b$ that transforms this ellipse into the unit circle.

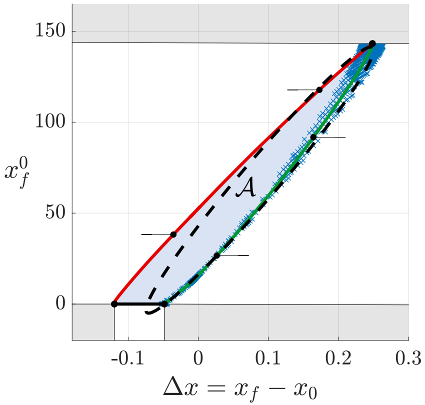


Figure: Original coordinates

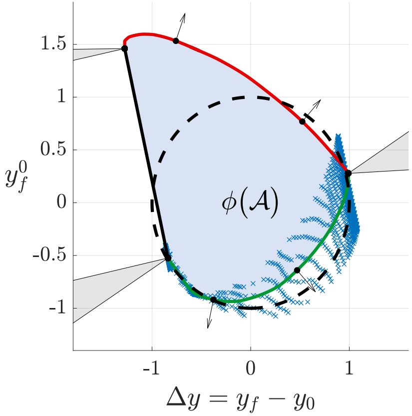
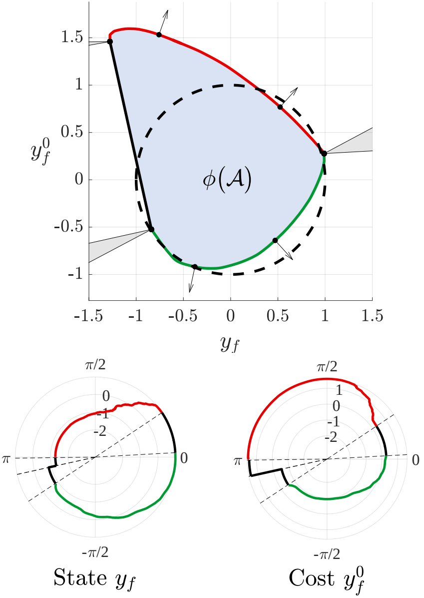


Figure: New coordinates

Geometric interpretation of the costate



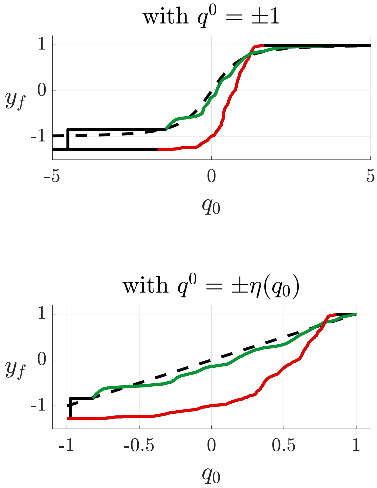


Figure: With $x_0 = 0.5$.

Definition of the shooting functions

In the new coordinates, the shooting function $T\colon \mathbb{R}^- \times \mathbb{R} \to \mathbb{R}$ is given by

$$T(\hat{q}_0) = \pi_y\big(\hat{y}_f(\hat{q}_0)\big) - y_T$$

where π_y is the classical y-space projection, and the function $\hat{y}_f(\cdot)$ is constructed by

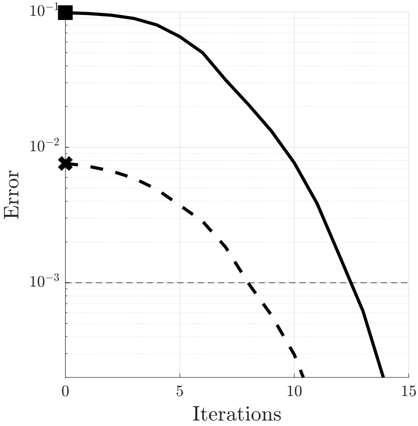
$$\begin{array}{ccc}
(p^0, p_0) = \hat{p}_0 & \xleftarrow[\hat{p}_0 = A^\top \hat{q}_0]{\hat{p}_0 = J_\phi(\hat{x}_0)^\top \hat{q}_0} & \hat{q}_0 = (q^0, q_0) \\
\hat{x}_f = \pi_{\hat{x}}\big(\exp_{\vec{h}}(\hat{x}_0, \hat{p}_0)\big) \Big\downarrow & & \\
(x_f^0, x_f) = \hat{x}_f & \xrightarrow[A x_f + b = \hat{y}_f]{\phi(\hat{x}_f) = \hat{y}_f} & \hat{y}_f = (y_f^0, y_f)
\end{array}$$

The functions T_1 and T_2 are defined from T similarly as S_1 and S_2 from S.

Results

Figure: Evolution of the error w.r.t the number of iterations (with 100 different initial and final states).

Error \ Init	Fixed[4]	Natural
	■	✖
$\lvert S_1(\cdot) \rvert$	———	- - -

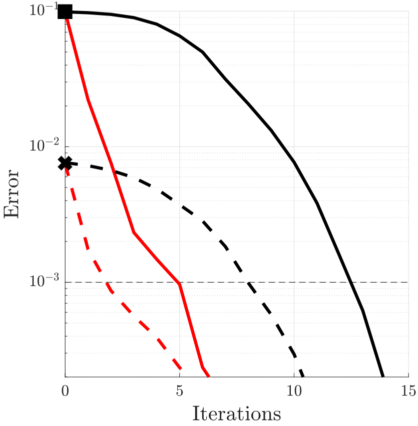


[4] $p = 500$ for S_1

Results

Figure: Evolution of the error w.r.t the number of iterations
(with 100 different initial and final states).

Init / Error	Fixed[4]	Natural
	■	✖
$\lvert S_1(\cdot) \rvert$	—	- - -
for T_2	—	- - -



The error for T_2 is converted into the original coordinates.

[4] $p = 500$ for S_1 and $q = 0$ for T_2.

Conclusion

We propose a new geometric preconditioner of the shooting function :

- based on a geometric interpretation of the costate and on the Mathieu transformation,
- which only needs 2 iterations in average of the solver to find a zero in our application,
- which is non-intrusive with respect to the model,
- no additional computational cost (since we have $\mathbb{D}$ and C).

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Main property on the transformation

If $\phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphism then

$$\left\{\begin{array}{l} \min\limits_{\hat{x}=(x^0,x)} \; x^0, \\ \text{s.t. } \hat{x} \in \mathcal{A}, \\ \qquad x = x_T, \end{array}\right. \iff \left\{\begin{array}{l} \min\limits_{\hat{y}=(y^0,y)} \; \pi_{x^0}\big(\phi^{-1}(\hat{y})\big), \\ \text{s.t. } \hat{y} \in \phi\big(\mathcal{A}\big), \\ \qquad \pi_x\big(\phi^{-1}(\hat{y})\big) = x_T, \end{array}\right.$$

where π_{x^0} is the x^0-space projection. Moreover, if ϕ satisfy

$$\frac{\partial \phi}{\partial x^0} = \begin{pmatrix} k \\ 0 \end{pmatrix}, \quad k > 0, \tag{3}$$

then $\phi(\hat{x}) = \big(\phi_0(\hat{x}), \phi_x(x)\big)$ and

$$\left\{\begin{array}{l} \min\limits_{\hat{x}=(x^0,x)} \; x^0, \\ \text{s.t. } \hat{x} \in \mathcal{A}, \\ \qquad x = x_f, \end{array}\right. \iff \left\{\begin{array}{l} \min\limits_{\hat{y}=(y^0,y)} \; y^0, \\ \text{s.t. } \hat{y} \in \phi\big(\mathcal{A}\big), \\ \qquad y = y_T, \end{array}\right.$$

where $y_T = \phi_x(x_T)$.

Results

Figure: Evolution of the error

Black: $|S_1(\cdot)|$

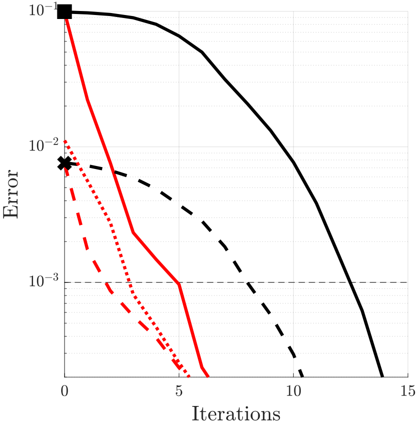
Red: for $T_2(\cdot)$

w.r.t. the number of iterations (with 100 different final states).

—— / ——: fixed init (■) ($p = 500$ / $q = 0$)

- - - / - - -: natural init (✖)

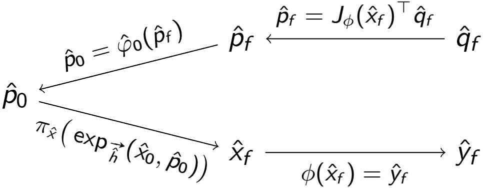
····· : $q = y_T$



The error for T_2 is calculated for each iterate on the initial coordinates.

Definition in the general case

In a general case, the function $\hat{y}_f(\cdot)$ is constructed by



where the function $\hat{\varphi}_0$ is an approximation of the map $\hat{p}_f \mapsto \hat{p}_0$. In our case, this approximation is the identity: