Geometric preconditioner for indirect shooting and application to hybrid vehicle

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Introduction

In collaboration with:

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- Sophie Jan, IMT, Toulouse,
- Serge Laporte, IMT, Toulouse,
- Mariano Sans, Vitesco Technologies, Toulouse.

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Cycle

We consider a Hybrid Electric Vehicle (HEV) on a predefined cycle, i.e. speed and slope trajectories are prescribed.
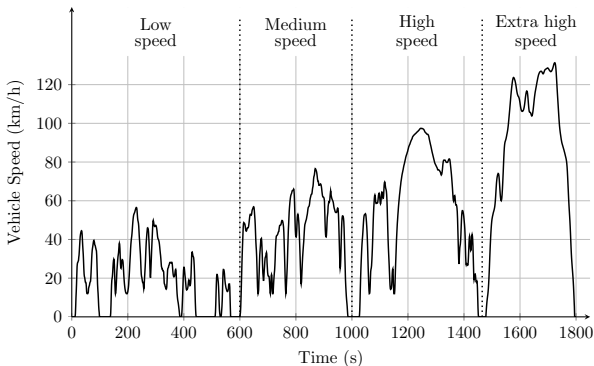

Figure: Worldwide harmonized Light vehicles Test Cycle (WLTC).

Requested wheels torque $T_{qW}(t)$ and rotation speed $N_W(t)$ are obtained with the information of our vehicle (mass, wheel diameter, aerodynamic coefficient...).

Static model of HEV

Inputs of our static model:

Name	Description	Unit
Cost		
m_{Fuel}	Fuel consumption	g
State		
SOC	Battery state of charge	
Commands		
Gear	Gearbox selector	
T_{qICE}	ICE torque	N.m
External inputs		
T_{qW}	Wheels torque	N.m
N_W	Wheels rotation speed	RPM
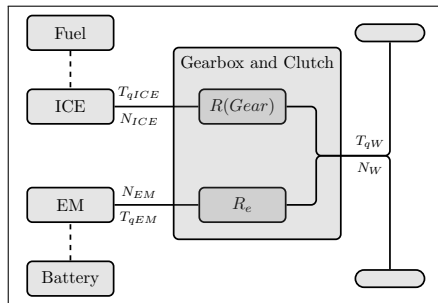
Outputs: $\dot{m}_{Fuel}$ and $\dot{SOC}$, where $\dot{}$ stands for $\dfrac{\mathrm{d}}{\mathrm{d}t}$.

HEV torque split and gear shift problem

The HEV torque split and gear shift problem can be formulated as a classical Lagrange optimal control problem

$$
\text{(OCP)} \quad
\begin{cases}
V(x_0, x_T) = \min\limits_{x,u} \int_{t_0}^{t_f} f^0\big(t, x(t), u(t)\big)\, \mathrm{d}t, \\[2mm]
\text{s.t. } \dot{x}(t) = f\big(t, x(t), u(t)\big), & t \in [t_0, t_f] \text{ a.e.}, \\[1mm]
\qquad u(t) \in \mathrm{U}(t), & t \in [t_0, t_f], \\[1mm]
\qquad x(t_0) = x_0, \quad x(t_f) = x_T,
\end{cases}
$$

where:

- $x \in \mathrm{AC}([t_0, t_f], \mathbb{R})$ corresponds to the *SOC*,
- $u \in \mathrm{L}^\infty([t_0, t_f], \mathbb{R}^2)$ corresponds to the pair $\big(T_{qICE}, Gear\big)$,
- functions f^0 and f are $\mathcal{C}^1$ w.r.t. x and u,
- $\mathrm{U}(t) \subset \mathbb{R}^2$ is a nonempty closed set for every $t \in [t_0, t_f]$, with regularity assumptions.[1]

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[1](cf. [Cesari, 1983, Chapter 4.2, Remark 5] for more information)

Assumptions from previous work

Motivated by [Cots et al., 2023a],

- we only consider the first 100s of the cycle ($[t_0, t_f] = [0, 100]$),
- we have constructed a database $\mathbb{D}$ of the value function V evaluations by an efficient method[2],
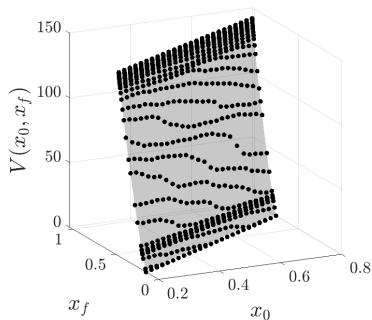- we have trained a neural network C on $\mathbb{D}$ to approximate V.

Figure: The points correspond to $\mathbb{D}$ and the surface to the trained neural network C.

[2]cf. [Cots et al., 2023b] for more information

Augmented system

We propose to consider the augmented formulation of (OCP)

(AOCP)
$$
\begin{cases}
\min\limits_{\hat{x}, u} x^0(t_f) \\
\text{s.t. } \dot{\hat{x}}(t) = \hat{f}(t, \hat{x}(t), u(t)) & t \in [t_0, t_f] \text{ a.e.}, \\
\quad u(t) \in \mathrm{U}(t) & t \in [t_0, t_f], \\
\quad \hat{x}(t_0) = \hat{x}_0, \quad x(t_f) = x_f,
\end{cases}
$$

where $\hat{f} \colon \mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is the augmented system

$$
\hat{f}(t, \hat{x}, u) = \left(f^0(t, x, u), f(t, x, u) \right)
$$

and where $\hat{x} = (x^0, x)$ corresponds to the cost-state pair, with $\hat{x}_0 = (0, x_0)$.

Pontryagin's Maximum Principle

If $(\hat{x}, u)$ is a solution of (AOCP) then it is a projection of an augmented extremal $(\hat{x}, \hat{p}, u)$ where $\hat{p} = (p^0, p) \in \text{AC}([t_0, t_f], \mathbb{R}^2) \neq 0$ is the non-trivial augmented costate, with $p^0 \leq 0$, and which satisfies the Hamilton's dynamic for almost every $t \in [t_0, t_f]$

$$\begin{cases} \dot{\hat{x}}(t) = \nabla_{\hat{p}} h(t, \hat{x}(t), \hat{p}(t), u(t)), \\ \dot{\hat{p}}(t) = -\nabla_{\hat{x}} h(t, \hat{x}(t), \hat{p}(t), u(t)), \end{cases} \quad (1)$$

as well as the maximization condition for almost every $t \in [t_0, t_f]$

$$h(t, \hat{x}(t), \hat{p}(t), u(t)) = \max_{w \in U(t)} h(t, \hat{x}(t), \hat{p}(t), w), \quad (2)$$

where $h(t, \hat{x}, \hat{p}, u) = (\hat{p} \mid \hat{f}(t, \hat{x}, u))$ is the pseudo-Hamiltonian of the augmented system.

Notations

For the following presentation, we denote

$$\hat{x} = (x^0, x) \quad \text{and} \quad \hat{p} = (p^0, p).$$

Moreover, we denote

$$\hat{z} = (\hat{x}, \hat{p}) \quad \text{and} \quad z = (x, p).$$

These notations can be used for absolutely continuous functions or for vectors.

Remark

Since $\hat{f}$ does not depend on the cost x^0, we obtain $\dot{p}^0(\cdot) = 0$ and thus $p^0(\cdot)$ is constant.

Framework

Let us denote $\exp_{\overrightarrow{h}}(\hat{z}_0)$ a solution at time t_f of

$$
\begin{cases}
\dot{\hat{z}}(t) = \overrightarrow{h}\big(t, \hat{z}(t), u(t)\big), & t \in [t_0, t_f] \text{ a.e.} \\
h\big(t, \hat{z}(t), u(t)\big) = \max_{w \in U(t)} h\big(t, \hat{z}(t), w\big), & t \in [t_0, t_f] \text{ a.e.} \\
\hat{z}(t_0) = \hat{z}_0,
\end{cases}
$$

where $\overrightarrow{h}$ is the pseudo-Hamiltonian vector field of the augmented system, defined by

$$
\overrightarrow{h}(t, \hat{x}, \hat{p}, u) = \big(\nabla_{\hat{x}} h(t, \hat{x}, \hat{p}, u), -\nabla_{\hat{p}} h(t, \hat{x}, \hat{p}, u)\big).
$$

Hypothesis 1

The possibly multivalued function $\exp_{\overrightarrow{h}}(\hat{x}_0, \hat{p}_0)$ is an application, defined for all $\hat{x}_0 \in \mathbb{R}^2$ and for all non trivial $\hat{p}_0 \in \mathbb{R}^2$.

Simple shooting method

Under the previous hypothesis, the maximum principle leads to the resolution of

$$(\text{TPBVP}) \qquad \begin{cases} \pi_x\big(\exp_{\overrightarrow{h}}(\hat{z}_0)\big) = x_T, \\ \pi_{\hat{x}}(\hat{z}_0) = \hat{x}_0, \quad \pi_{p^0}(\hat{z}_0) \leq 0, \end{cases}$$

where $\pi_x(\cdot)$ is the classical x-space projection.

The simple shooting method aims to find a non-trivial zero $\hat{p}_0 = (p^0, p_0)$ of the shooting function

$$\begin{array}{rccc} S & : & \mathbb{R}^- \times \mathbb{R} & \longrightarrow & \mathbb{R} \\ & & \hat{p}_0 & \longmapsto & \pi_x\big(\exp_{\overrightarrow{h}}(\hat{x}_0, \hat{p}_0)\big) - x_T \end{array}$$

Normalization of the shooting function

Let us remark that if $\hat{p}_0 \neq 0$ satisfies $S(\hat{p}_0) = 0$ then for all $k > 0$, $S(k\hat{p}_0) = 0$ (due to homogeneity of BC-extremals on $\hat{p}$).

We propose two normalizations of the shooting function S.

- <u>Method 1:</u> if we assume that the extremals associated to a solution are normal ($p^0 < 0$), then we can fix $p^0 = -1$ and consider $S_1 \colon \mathbb{R} \to \mathbb{R}$ defined by

$$S_1(p_0) = S(-1, p_0),$$

- <u>Method 2:</u> without the above assumption, we can fix $\|\hat{p}_0\|_2 = 1$ and consider $S_2 \colon [-1, 1] \to \mathbb{R}$ defined by

$$S_2(p_0) = S\big(\eta(p_0), p_0\big), \quad \text{where} \quad \eta(p_0) = -\sqrt{1 - p_0^2}.$$

Results

A solution of the shooting method is found by a Newton-like solver.

Thanks to [Cots et al., 2023a], a natural initial guess for S_1 is given by

$$p_* = -\nabla_{x_0} C(x_0, x_f).$$
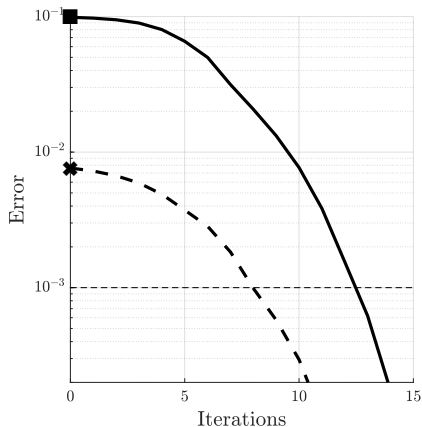
Figure: Evolution of the error $|S_1(\cdot) - x_f|$ w.r.t the number of iterations (with 100 different initial and final states).
—: fixed initialization $p = 500$ (■)   - - -: natural initialization p_* (✖)
- - -: industrial tolerance 10^{-3}

Goal: Reducing the number of iterations of the solver

Main idea[3]: Preconditioning method of the shooting function based on
- a geometric interpretation of the costate
- and the Mathieu transformation.

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[3]cf. [Cots et al., 2024] for more information

Geometric interpretation of the costate

The proof of the maximum principle is constructive.
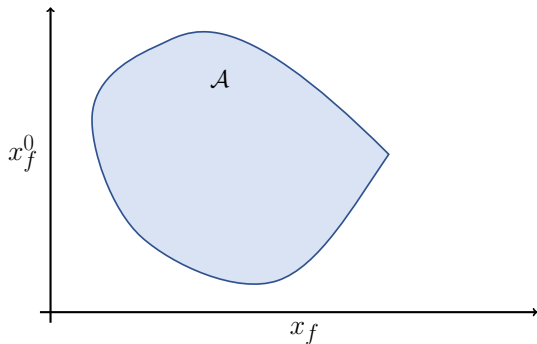
Figure: Illustration of the accessible augmented state set $\mathcal{A}$, which is the set of reachable augmented states $\hat{x}_f = (x_f^0, x_f)$ at t_f from $\hat{x}_0$ at t_0.

The proof of the maximum principle is constructive. The final augmented costate $\hat{p}_f = \left(p^0, p(t_f)\right)$ is taken in the polar of the proper convex Boltyanskii cone $\mathcal{K}^\circ$.
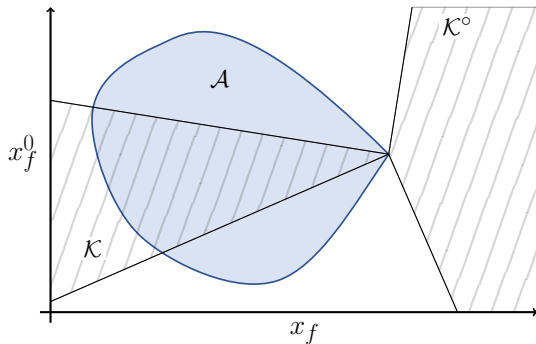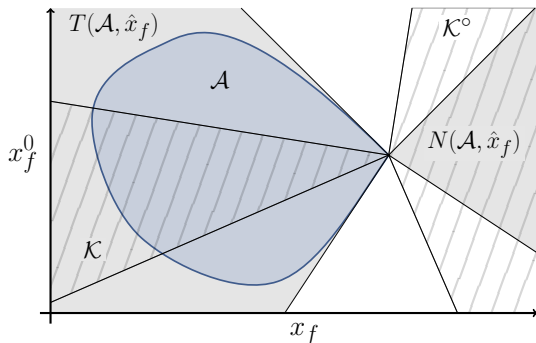

Figure: Illustration of the Botlyanskii cone $\mathcal{K}$ and its polar $\mathcal{K}^\circ$ at an augmented final state $\hat{x}_f \in \partial\mathcal{A}$.

Geometric interpretation of the costate

The proof of the maximum principle is constructive. The final augmented costate $\hat{p}_f = \left(p^0, p(t_f)\right)$ is taken in the polar of the proper convex Boltyanskii cone $\mathcal{K}^\circ$.

Figure: Illustration of the link between $\mathcal{K}^\circ$ and the normal cone $N(\mathcal{A}, \hat{x}_f)$ of the set $\mathcal{A}$ at the point $\hat{x}_f$.

If $\mathcal{A}$ is closed and convex, we can take $\hat{p}(t_f) \in N(\mathcal{A}, \hat{x}_f)$.

with $p^0 = \pm 1$

with $p^0 = \pm \eta(p_0)$

State x_f

Cost x_f^0
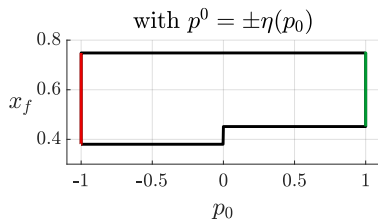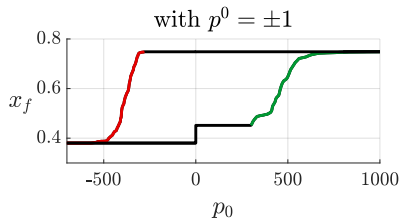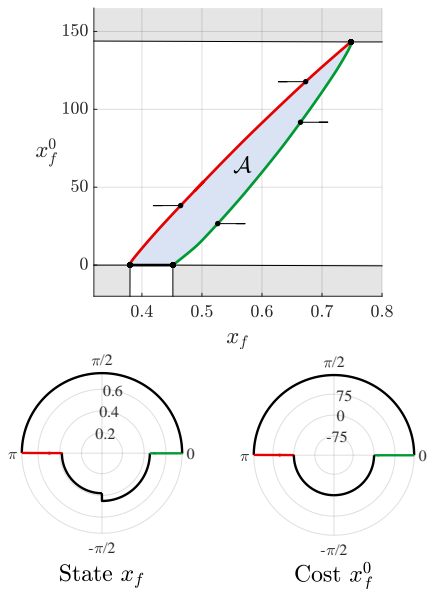
Figure: With $x_0 = 0.5$.

Mathieu transformation

A diffeomorphism $\phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ on the augmented state is lifted into a diffeomorphism $\Phi\colon \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2 \times \mathbb{R}^2$ on the augmented state-costate that preserves the Hamiltonian dynamics

$$\Phi(\hat{x}, \hat{p}) = \big(\phi(\hat{x}), J_\phi(\hat{x})^{-\top}\hat{p}\big),$$

which is called Mathieu transformation.

This diffeomorphism transforms $\hat{z} = (\hat{x}, \hat{p})$ into $\hat{w} = (\hat{y}, \hat{q})$:

$$\hat{z} = \left(\begin{array}{c} \hat{x} \\ \hat{p} \end{array} \right) \xrightarrow[\Phi^{-1}]{\Phi} \left(\begin{array}{c} \hat{y} \\ \hat{q} \end{array} \right) = \hat{w}.$$

Moreover, we denote $\hat{y} = (y^0, y)$ and $\hat{q} = (q^0, q)$.

Construction of the transformation

Main idea: fitting an ellipse on $\partial\mathcal{A}$ and creating the linear diffeomorphism $\phi(\hat{x}) = A\hat{x} + b$ that transforms this ellipse into the unit circle.
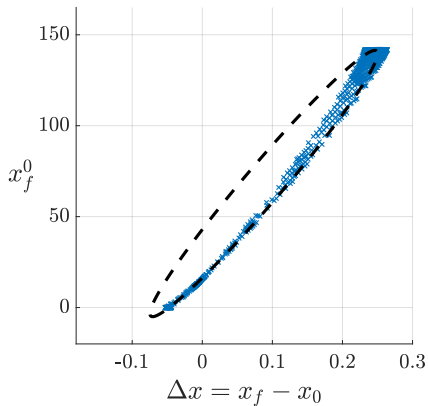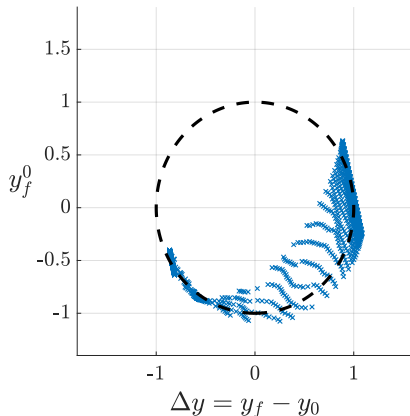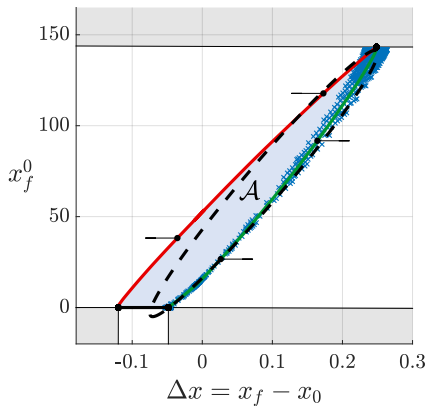

Figure: Original coordinates

Figure: New coordinates

Construction of the transformation

Main idea: fitting an ellipse on $\partial \mathcal{A}$ and creating the linear diffeomorphism $\phi(\hat{x}) = A\hat{x} + b$ that transforms this ellipse into the unit circle.
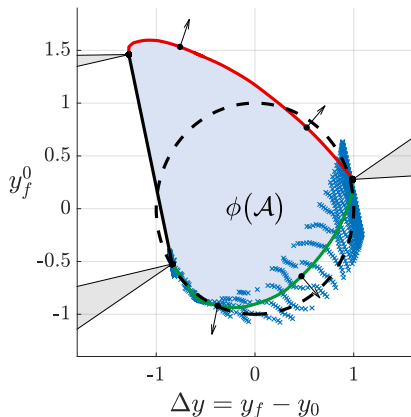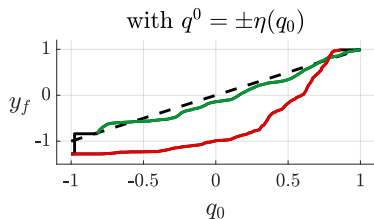

Figure: Original coordinates

Figure: New coordinates

with $q^0 = \pm 1$

with $q^0 = \pm \eta(q_0)$

Figure: With $x_0 = 0.5$.

State y_f

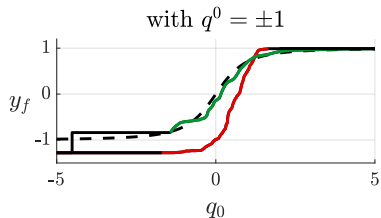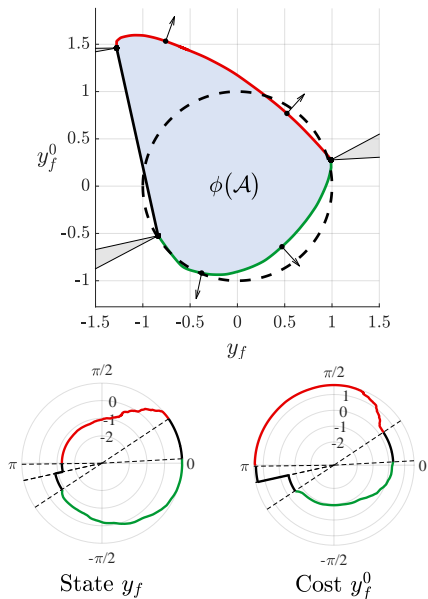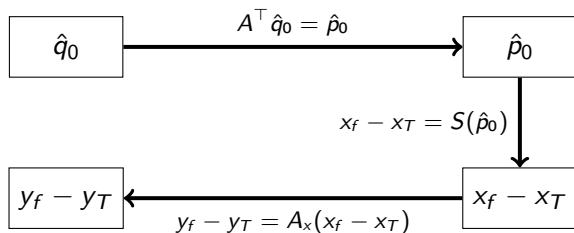Cost y_f^0

Definition of the shooting functions

In the new coordinates, the shooting function $T \colon \mathbb{R}^- \times \mathbb{R} \to \mathbb{R}$ is given by

$$T(\hat{q}_0) = A_x S(A^\top \hat{q}_0),$$

where

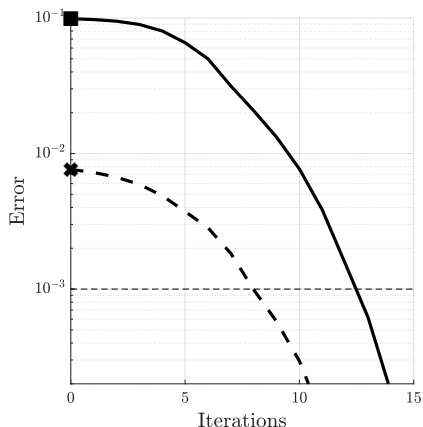$$A = \begin{bmatrix} k & A_{x^0} \\ 0 & A_x \end{bmatrix}$$

with $k > 0$. The shooting function T is schematized by

where $y_T \coloneqq A_x x_T$. The functions T_1 and T_2 are defined from T similarly as S_1 and S_2 from S.

Results

Figure: Evolution of the error w.r.t the number of iterations (with 100 different initial and final states).

	Init	Fixed[4]	Natural
Error		■	✖
$\left|S_1(\cdot)\right|$		—	- - -

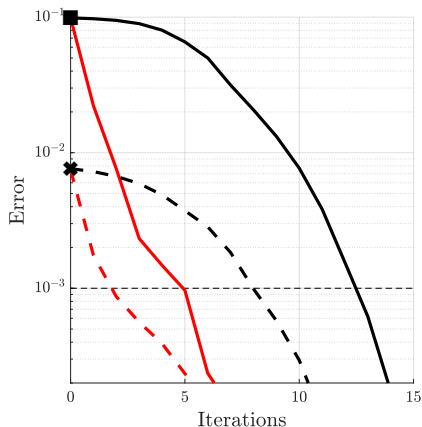[4] $p = 500$ for S_1

Results

Figure: Evolution of the error w.r.t the number of iterations (with 100 different initial and final states).

	Init	Fixed[4]	Natural
Error		■	✖
$\left|S_1(\cdot)\right|$		——	- - -
for T_2		——	- - -

The error for T_2 is converted into the original coordinates.

---
[4] $p = 500$ for S_1 and $q = 0$ for T_2.

Conclusion

We propose a new geometric preconditioner of the shooting function :

- based on a geometric interpretation of the costate and on the Mathieu transformation,
- which only needs 2 iterations in average of the solver to find a zero in our application,
- which is non-intrusive with respect to the model,
- no additional computational cost (since we have $\mathbb{D}$ and C).

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Geometric preconditioner for indirect shooting and application to hybrid vehicle.
Proceeding submitted to the IFAC MICNON 2024 conference.

Main property on the transformation

If $\phi \colon \mathbb{R}^2 \to \mathbb{R}^2$ is a diffeomorphism then

$$\left\{ \begin{array}{c} \min_{\hat{x}=(x^0,x)} x^0, \\ \text{s.t. } \hat{x} \in \mathcal{A}, \\ x = x_T, \end{array} \right. \iff \left\{ \begin{array}{c} \min_{\hat{y}=(y^0,y)} \pi_{x^0}\big(\phi^{-1}(\hat{y})\big), \\ \text{s.t. } \hat{y} \in \phi(\mathcal{A}), \\ \pi_x\big(\phi^{-1}(\hat{y})\big) = x_T, \end{array} \right.$$

where π_{x^0} is the x^0-space projection. Moreover, if ϕ satisfy

$$\frac{\partial \phi}{\partial x^0} = \left(\begin{array}{c} k \\ 0 \end{array} \right), \quad k > 0, \tag{3}$$

then $\phi(\hat{x}) = \big(\phi_0(x^0), \phi_x(x)\big)$ and

$$\left\{ \begin{array}{c} \min_{\hat{x}=(x^0,x)} x^0, \\ \text{s.t. } \hat{x} \in \mathcal{A}, \\ x = x_f, \end{array} \right. \iff \left\{ \begin{array}{c} \min_{\hat{y}=(y^0,y)} y^0, \\ \text{s.t. } \hat{y} \in \phi(\mathcal{A}), \\ y = y_T, \end{array} \right.$$

where $y_T = \phi_x(x_T)$.

Results
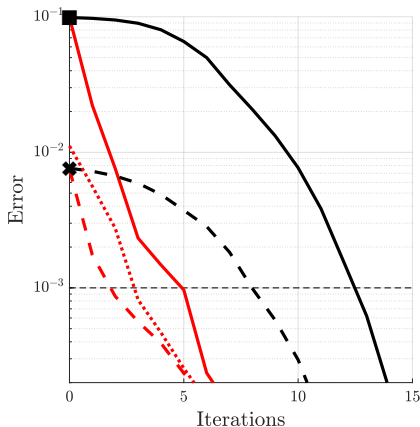
Figure: Evolution of the error

Black: $|S_1(\cdot)|$

Red: for $T_2(\cdot)$

w.r.t. the number of iterations
(with 100 different final states).

— / —: fixed init (■)
($p = 500$ / $q = 0$)

--- / ---: natural init (✱)

······ : $q = y_T$

The error for T_2 is calculated for each iterate on the initial coordinates.

Definition in the general case

In a general case, the function $\hat{y}_f(\cdot)$ is constructed by

$$\hat{p}_0 \xleftarrow{\hat{p}_0 = \hat{\varphi}_0(\hat{p}_f)} \hat{p}_f \xleftarrow{\hat{p}_f = J_\phi(\hat{x}_f)^\top \hat{q}_f} \hat{q}_f$$

$$\hat{p}_0 \xrightarrow{\pi_{\hat{x}}\left(\exp_{\vec{h}}(\hat{x}_0, \hat{p}_0)\right)} \hat{x}_f \xrightarrow{\phi(\hat{x}_f) = \hat{y}_f} \hat{y}_f$$

where the function $\hat{\varphi}_0$ is an approximation of the map $\hat{p}_f \mapsto \hat{p}_0$. In our case, this approximation is the identity: