Optimal synthesis of membrane filtration systems

A Julia package to improve performances by optimal control

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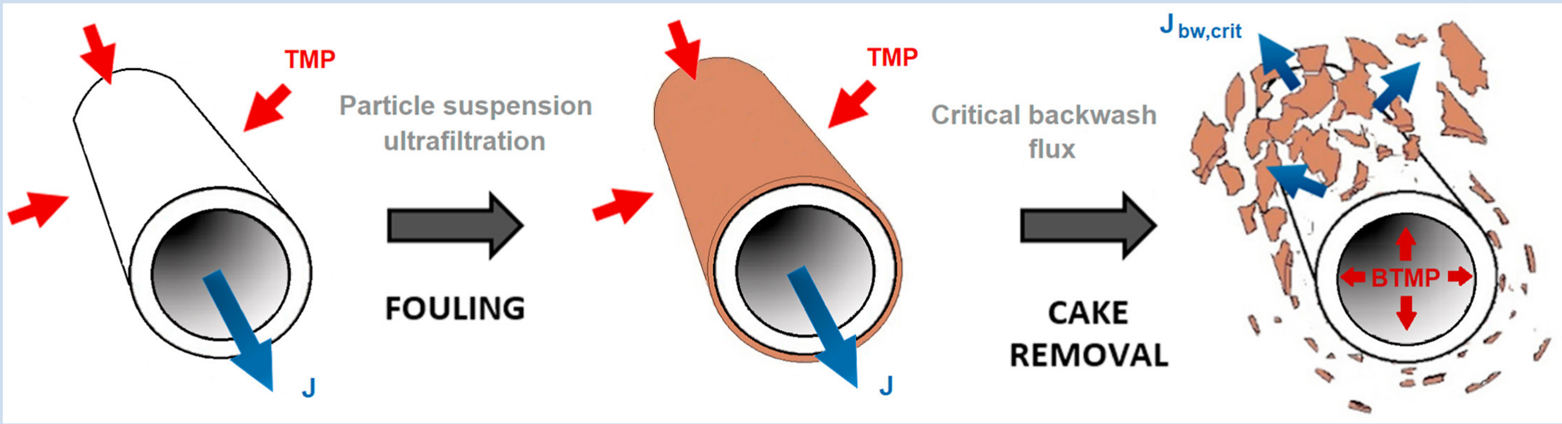
Introduction

Filtration.jl package aims to provide an optimal control framework to control membrane filtration systems in order to maximize its efficiency (max volume, min energy, ...). It generates automatically a feedback controller for a large class of problems, where the main functions are user-defined. This package mainly use combination of automatic differentiation (**ForwardDiff.jl**) and resolution of ODE (**OrdinaryDiffEq.jl**).

Membrane filtration system

Membrane filtration systems have to alternate between filtration and backwash phases.



The system is composed by one internal resistance, denoted x_2 and two outputs :

- x^0 has to be minimized at the final time t_f,
- x_1 which has to reach a target value T at terminal time t_f.

The control $u \in [-1, 1]$ represents the mode of the system : $u = +1$ corresponds to filtration and $u = -1$ to backwash. Assuming that the dynamic of $x = (x^0, x_1, x_2)$ is user-defined by $f(x_2)$ and $g(x_2)$ respectively in filtration and backwash mode, we are interested in solving

$$\begin{cases} \min\limits_{x,u} x^0(t_f) \\ \text{s.t. } \dot{x}(t) = \dfrac{f(x_2(t)) + g(x_2(t))}{2} + u(t)\dfrac{f(x_2(t)) - g(x_2(t))}{2}, \\ \qquad u(t) \in [-1, 1], \qquad t \in [t_0, t_f] \text{ a.e.}, \\ \qquad x(t_0) = x_0, \end{cases}$$

where t_f is the first time such that $x_1(t_f) \geq T$.

Main theoretical results

Structure : Thanks to the Pontryagin Maximum Principle and the Green Theorem, under some conditions, the structure of optimal solution can only be one of σ_+, $\sigma_-\sigma_+$, $\sigma_s\sigma_+$, $\sigma_-\sigma_s\sigma_+$ or $\sigma_+\sigma_s\sigma_+$, where σ_+ is a bang $u = +1$ arc, σ_- is a bang $u = -1$ arc, and σ_s is a singular $x_2 = \bar{x}_2$ arc with the control

$$u = u_s(\bar{x}_2) = -\frac{f_2(\bar{x}_2) + g_2(\bar{x}_2)}{f_2(\bar{x}_2) - g_2(\bar{x}_2)}.$$

Feedback control : The optimal control can be given in a feedback form by

$$u(x_1, x_2) = \begin{cases} +1 & \text{if } (x_1, x_2) \in \Omega_+, \\ -1 & \text{if } (x_1, x_2) \in \Omega_-, \\ u_s(x_2) & \text{if } (x_1, x_2) \in \mathcal{S}. \end{cases}$$

where Ω_+ and Ω_- are two connected sets, separated by singular locus $\mathcal{S}$, switching locus $\mathcal{C}$ and dispersal locus $\mathcal{D}$.

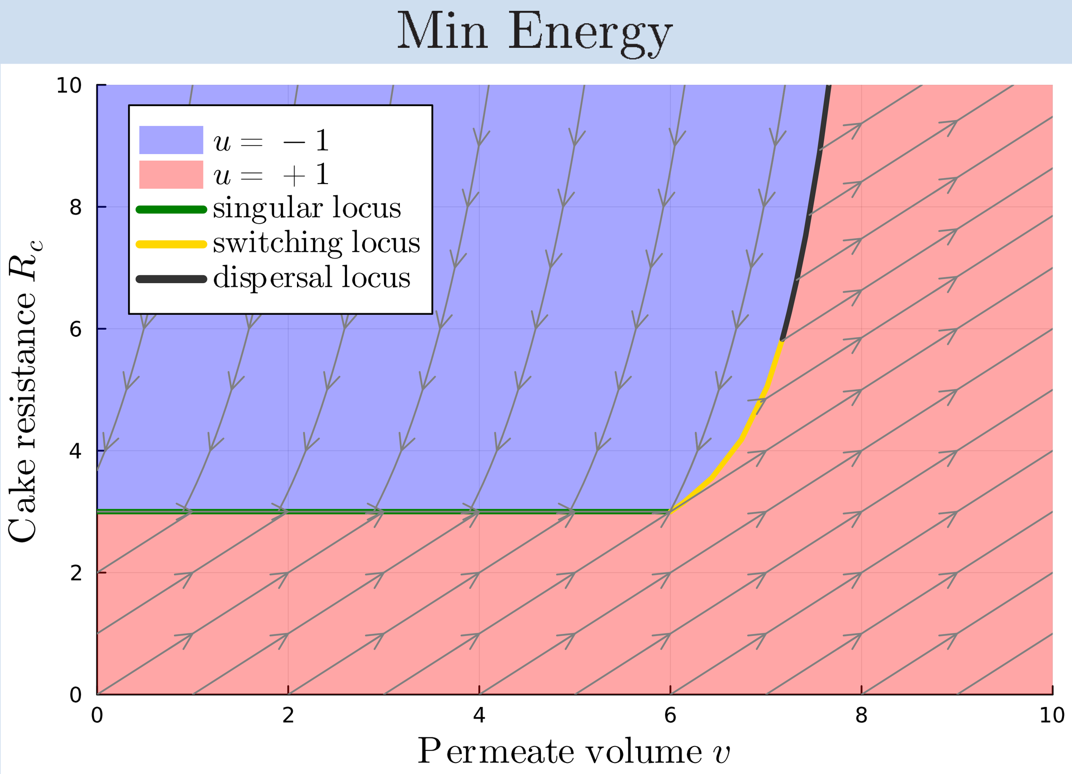
Differential continuation : One can characterize points $(x_1, x_2) \in \mathcal{C}$ (or $(x_1, x_2) \in \mathcal{D}$) as solution of $S(x_1, x_2) = 0$. By using the implicit function theorem, there exists a function ϕ such that $S(\phi(x_2), x_2) = 0$.
Function ϕ is thus solution of the following ODE

$$\phi'(x_2) = -\left(\frac{\partial S}{\partial x_1}(\phi(x_2), x_2)\right)^{-1} \frac{\partial S}{\partial x_2}(\phi(x_2), x_2), \quad \phi(\bar{x}_2) = \bar{x}_1,$$

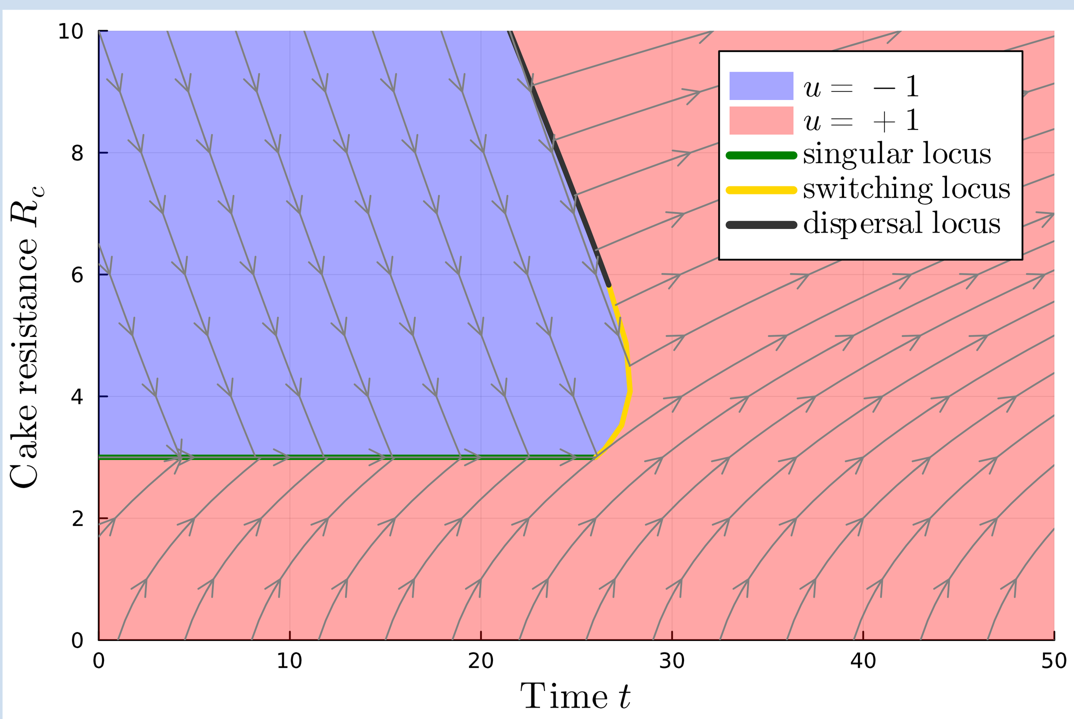
where $(\bar{x}_1, \bar{x}_2)$ is a known initial point.

Feedback synthesis

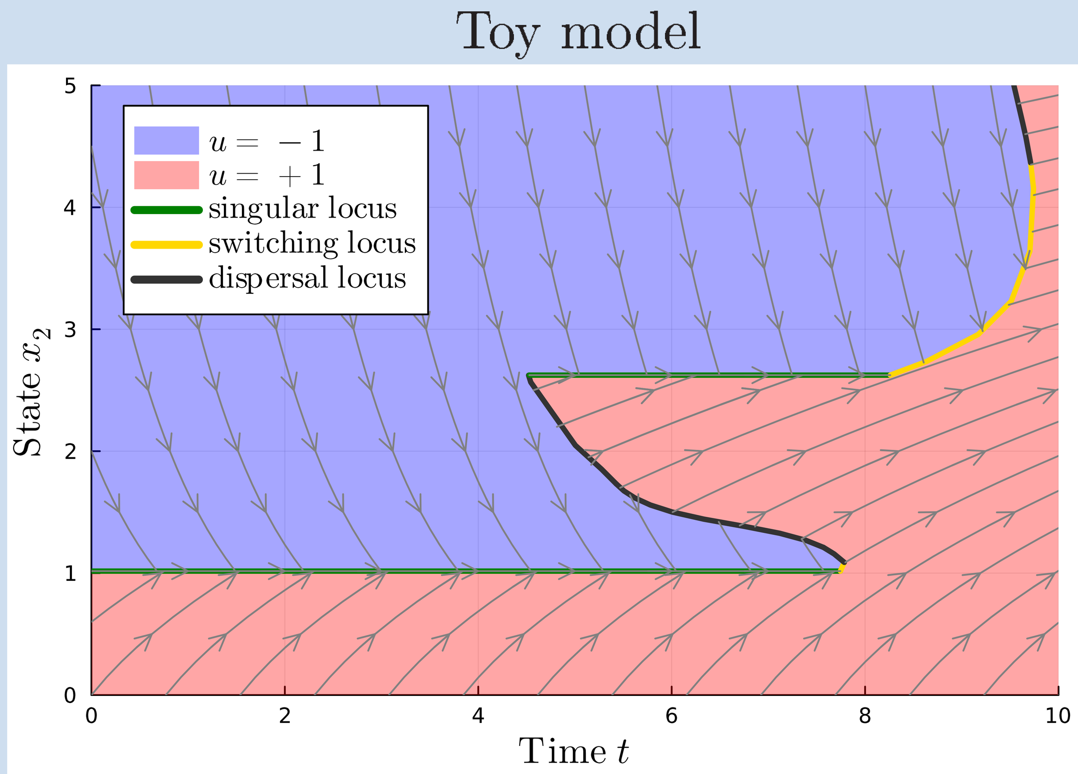
Min Energy



Max Volume



Toy model

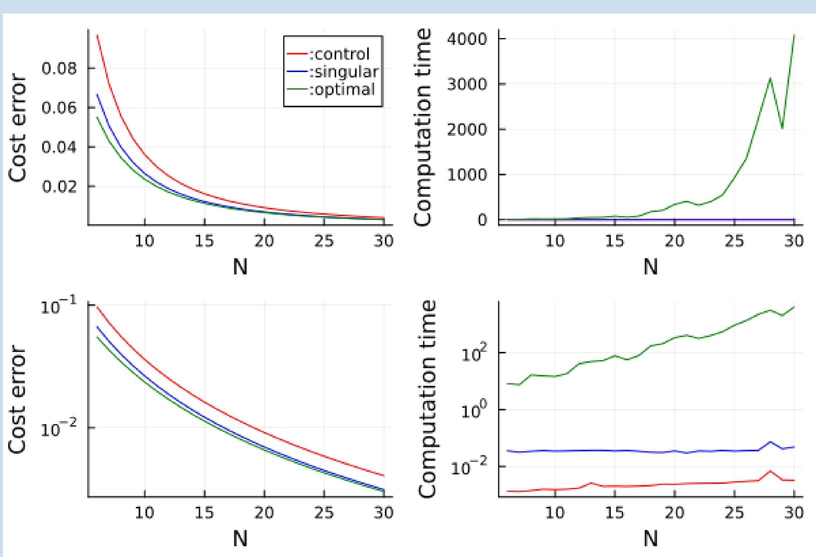


Operational control

When an optimal control has a singular arc, it must be approximated by a bang-bang arc in order to be implemented on a real membrane filtration system.

Given the number of commutation, three methods are proposed :

- approximation of control,
- approximation of state,
- global solution for the discretized problem



Conclusion & Perspectives

Filtration.jl provides automatic generation of optimal synthesis and delivers operational optimal control for membrane filtration systems.

Future works:

- use in real time the control generated by the package,
- fit model parameters to real data,
- study more complex problem (with two internal resistances for instance).

References

[1] Farouk Aichouche, Nesrine Kalboussi, Alain Rapaport, and Jérôme Harmand. Modeling and optimal control for production-regeneration systems - preliminary results -. In *2020 European Control Conference (ECC)*, 2020.

[2] Nesrine Kalboussi, Alain Rapaport, Térence Bayen, Nihel Ben Amar, Fatma Ellouze, and Jérome Harmand. Optimal control of membrane-filtration systems. *IEEE Transactions on Automatic Control*, 2019.